CHAPTER 1

Decision Analysis

CONTENTS

1.1 A Decision Tree Model and its Analysis
Bill Sampras' Summer Job Decision

1.2 Summary of the General Method of Decision Analysis

1.3 Another Decision Tree Model and its Analysis
Bio-Imaging Development Strategies

1.4 The Need for a Systematic Theory of Probability
Development of a New Consumer Product

1.5 Further Issues and Concluding Remarks on Decision Analysis

1.6 Case Modules
Kendall Crab and Lobster, Inc.
Buying a House
The Acquisition of DSOFT
National Realty Investment Corporation

1.7 Exercises

PERHAPS THE MOST FUNDAMENTAL AND IMPORTANT TASK THAT A MANAGER FACES is to make decisions in an uncertain environment. For example, a manufacturing manager must decide how much capital to invest in new plant capacity, when future demand for products is uncertain. A marketing manager must decide among a variety of different marketing strategies for a new product, when consumer response to these different marketing strategies is uncertain. An investment manager must decide whether or not to invest in a new venture, or whether or not to merge with another firm in another country, in the face of an uncertain economic and political environment.

In this chapter, we introduce a very important method for structuring and analyzing managerial decision problems in the face of uncertainty, in a systematic and rational manner. The method goes by the name **decision analysis.** The analytical model that is used in decision analysis is called a **decision tree.**

1.1 A DECISION TREE MODEL AND ITS ANALYSIS

Decision analysis is a logical and systematic way to address a wide variety of problems involving decision-making in an uncertain environment. We introduce the method of **decision analysis** and the analytical model of constructing and solving a **decision tree** with the following prototypical decision problem.

BILL SAMPRAS' SUMMER JOB DECISION

Bill Sampras is in the third week of his first semester at the Sloan School of Management at the Massachusetts Institute of Technology (MIT). In addition to spending time preparing for classes, Bill has begun to think seriously about summer employment for the next summer, and in particular about a decision he must make in the next several weeks.

On Bill's flight to Boston at the end of August, he sat next to and struck up an interesting conversation with Vanessa Parker, the Vice President for the Equity Desk of a major investment banking firm. At the end of the flight, Vanessa told Bill directly that she would like to discuss the possibility of hiring Bill for next summer, and that he should contact her directly in mid-November, when her firm starts their planning for summer hiring. Bill felt that she was sufficiently impressed with his experience (he worked in the Finance Department of a Fortune 500 company for four years on short-term investing of excess cash from revenue operations) as well as with his overall demeanor.

When Bill left the company in August to begin studying for his MBA, his boss, John Mason, had taken him aside and also promised him a summer job for the following summer. The summer salary would be $12,000 for twelve weeks back at the company. However, John also told him that the summer job offer would only be good until the end of October. Therefore, Bill must decide whether or not to accept John's summer job offer before he knows any details about Vanessa's potential job offer, as Vanessa had explained that her firm is unwilling to discuss summer job opportunities in detail until mid-November. If Bill were to turn down John's offer, Bill could either accept Vanessa's potential job offer (if it indeed were to materialize), or he could search for a different summer job by participating in the corporate summer recruiting program that the Sloan School of Management offers in January and February.

Bill's Decision Criterion

Let us suppose, for the sake of simplicity, that Bill feels that all summer job opportunities (working for John, working for Vanessa's firm, or obtaining a summer job through corporate recruiting at school) would offer Bill similar learning, networking, and resumé-building experiences. Therefore, we assume that Bill's only criterion on which to differentiate between summer jobs is the summer salary, and that Bill obviously prefers a higher salary to a lower salary.

Constructing a Decision Tree for Bill Sampras' Summer Job Decision Problem

A **decision tree** is a systematic way of organizing and representing the various decisions and uncertainties that a decision-maker faces. Here we construct such a decision tree for Bill Sampras' summer job decision.

Notice that there are, in fact, two decisions that Bill needs to make regarding the summer job problem. First, he must decide whether or not to accept John's summer job offer. Second, if he were to reject John's offer, and Vanessa's firm were to offer him a job in mid-November, he must then decide whether to accept Vanessa's offer or to instead participate in the school's corporate summer recruiting program in January and February.

These decisions are represented chronologically and in a systematic fashion in a drawing called a **decision tree.** Bill's first decision concerns whether to accept or reject John's offer. A decision is represented with a small box that is called a **decision node,** and each possible choice is represented as a line called a **branch** that emanates from the decision node. Therefore, Bill's first decision is represented as shown in Figure 1.1. It is customary to write a brief description of the decision choice on the top of each branch emanating from the decision node. Also, for future reference, we have given the node a label (in this case, the letter "A").

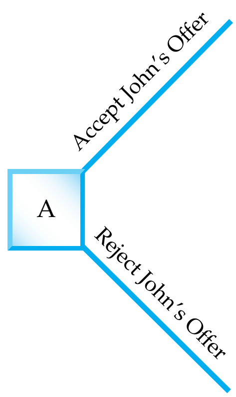
If Bill were to accept John's job offer, then there are no other decisions or uncertainties Bill would need to consider. However, if he were to reject John's job offer, then Bill would face the uncertainty of whether or not Vanessa's firm would subsequently offer Bill a summer job. In a decision tree, an uncertain event is represented with a small circle called an **event node,** and each possible outcome of the event is represented as a line (or branch) that emanates from the event node. Such an event node with its outcome branches is shown in Figure 1.2, and is given the label "B." Again, it is customary to write a brief description of the possible outcomes of the event above each outcome branch.

Unlike a decision node, where the decision-maker gets to select which branch to opt for, at an event node the decision-maker has no such choice. Rather, one can think that at an event node, "nature" or "fate" decides which outcome will take place.

The outcome branches that emanate from an event node must represent a **mutually exclusive** and **collectively exhaustive** set of possible events. By mutually exclusive, we mean that no two outcomes could ever transpire at the same time. By collectively exhaustive, we mean that the set of possible outcomes represents the entire range of possible outcomes. In other words, there is no probability that another non-represented outcome might occur. In our example, at this event node there are two, and only two, distinct outcomes that could occur: one outcome is that Vanessa's firm will offer Bill a summer job, and the other outcome is that Vanessa's firm will not offer Bill a summer job.

If Vanessa's firm were to make Bill a job offer, then Bill would subsequently have to decide to accept or to reject the firm's job offer. In this case, and if Bill were to accept the firm's job offer, then his summer job problem would be resolved. If Bill were to instead reject their offer, then Bill would then have to search for summer employment through the school's corporate summer recruiting program. The decision tree shown in Figure 1.3 represents these further possible eventualities, where the additional decision

FIGURE 1.1
Representation of a decision node.



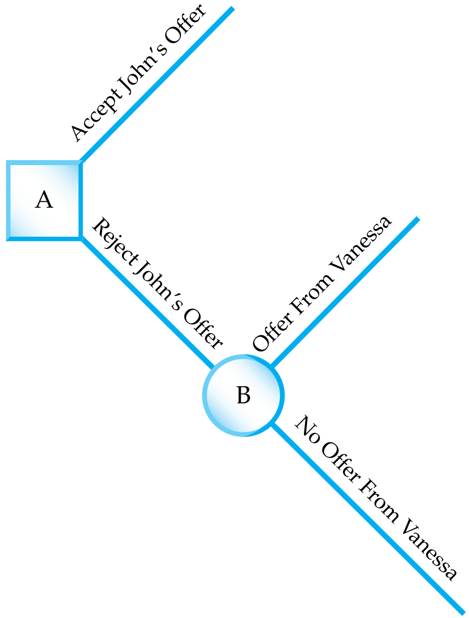


FIGURE 1.2
Representation of an event node.

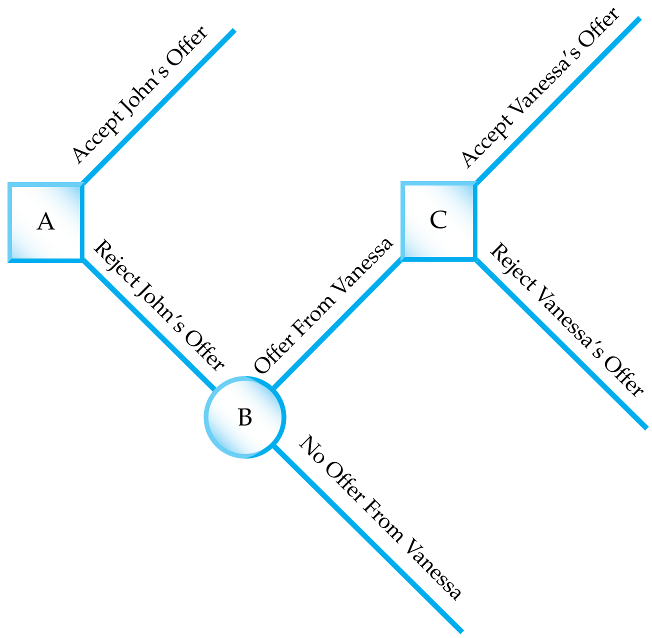


FIGURE 1.3
Further representation of the decision tree.

node C represents the decision that Bill would face if he were to receive a summer job offer from Vanessa's firm.

Assigning Probabilities

Another aspect of constructing a decision tree is the assignment or determination of the probability, i.e., the likelihood, that each of the various uncertain outcomes will transpire.

Let us suppose that Bill has visited the career services center at Sloan and has gathered some summary data on summer salaries received by the previous class of

MBA students. Based on salaries paid to Sloan students who worked in the Sales and Trading Departments at Vanessa's firm the previous summer, Bill has estimated that Vanessa's firm would make offers of $14,000 for twelve weeks' work to summer MBA students this coming summer.

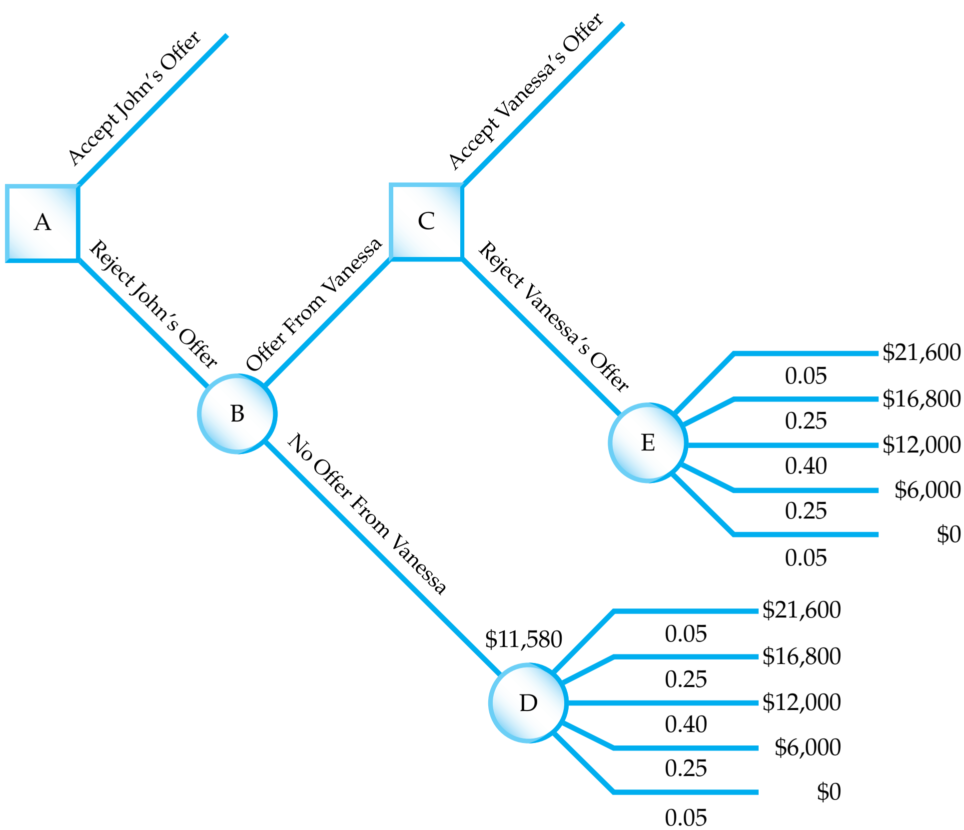
Let us also suppose that we have gathered some data on the salary range for all summer jobs that went to Sloan students last year, and that this data is conveniently summarized in Table 1.1. The table shows five different summer salaries (based on weekly salary) and the associated percentages of students who received this salary. (The school did not have salary information for 5% of the students. In order to be conservative, we assign these students a summer salary of $0.)

Suppose further that our own intuition has suggested that Table 1.1 is a good approximation of the likelihood that Bill would receive the indicated salaries if he were to participate in the school's corporate summer recruiting. That is, we estimate that there is roughly a 5% likelihood that Bill would be able to procure a summer job with a salary of $21,600, and that there is roughly a 25% likelihood that Bill would be able

TABLE 1.1
Distribution of summer salaries.

Weekly Salary	*Total Summer Pay (based on 12 weeks)*	*Percentage of Students Who Received This Salary*
$1,800	$21,600	5%
$1,400	$16,800	25%
$1,000	$12,000	40%
$500	$6,000	25%
$0	$0	5%

FIGURE 1.4
Further representation of the decision tree.



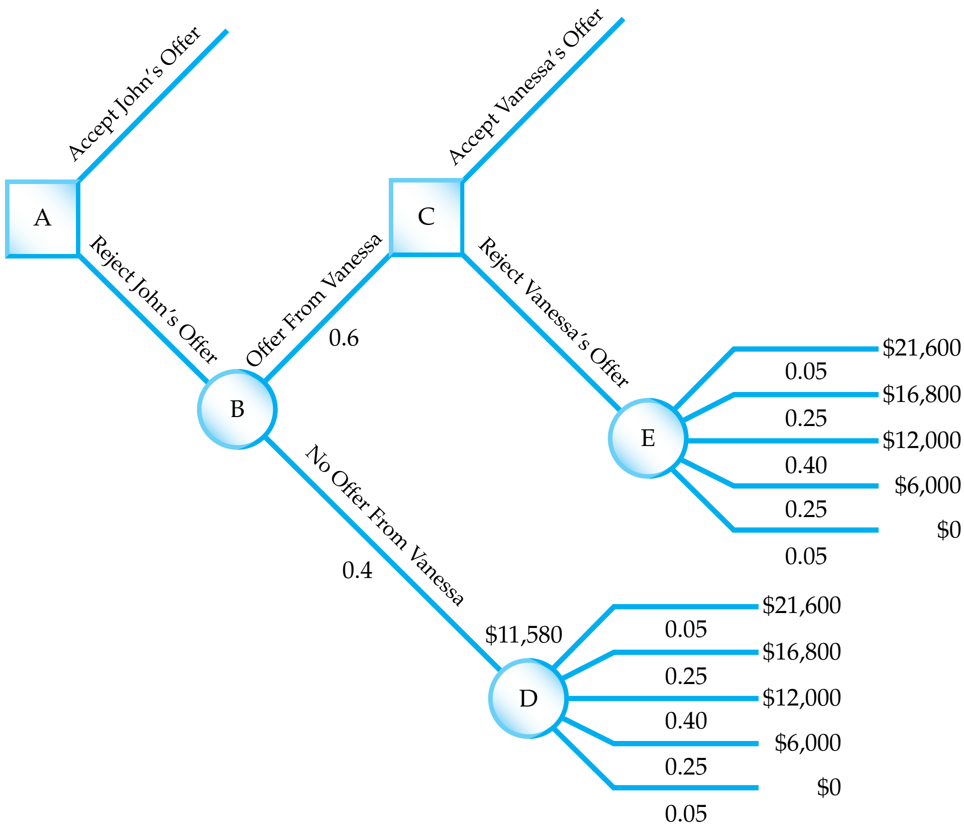
to procure a summer job with a salary of $16,800, etc. The now-expanded decision tree for the problem is shown in Figure 1.4, which includes event nodes D and E for the eventuality that Bill would participate in corporate summer recruiting if he were not to receive a job offer from Vanessa's firm, or if he were to reject an offer from Vanessa's firm. It is customary to write the probabilities of the various outcomes underneath their respective outcome branches, as is done in the figure.

Finally, let us estimate the likelihood that Vanessa's firm will offer Bill a job. Without much thought, we might assign this outcome a probability of 0.50, that is, there is a 50% likelihood that Vanessa's firm would offer Bill a summer job. On further reflection, we know that Vanessa was very impressed with Bill, and she sounded certain that she wanted to hire him. However, very many of Bill's classmates are also very talented (like him), and Bill has heard that competition for investment banking jobs is in fact very intense. Based on these musings, let us assign the probability that Bill would receive a summer job offer from Vanessa's firm to be 0.60. Therefore, the likelihood that Bill would not receive a job offer from Vanessa's firm would then be 0.40. These two numbers are shown in the decision tree in Figure 1.5.

Valuing the Final Branches

The next step in the decision analysis modeling methodology is to assign numerical values to the outcomes associated with the "final" branches of the decision tree, based on the decision criterion that has been adopted. As discussed earlier, Bill's decision criterion is his salary. Therefore, we assign the salary implication of each final branch and write this down to the right of the final branch, as shown in Figure 1.6.

FIGURE 1.5
Further representation of the decision tree.



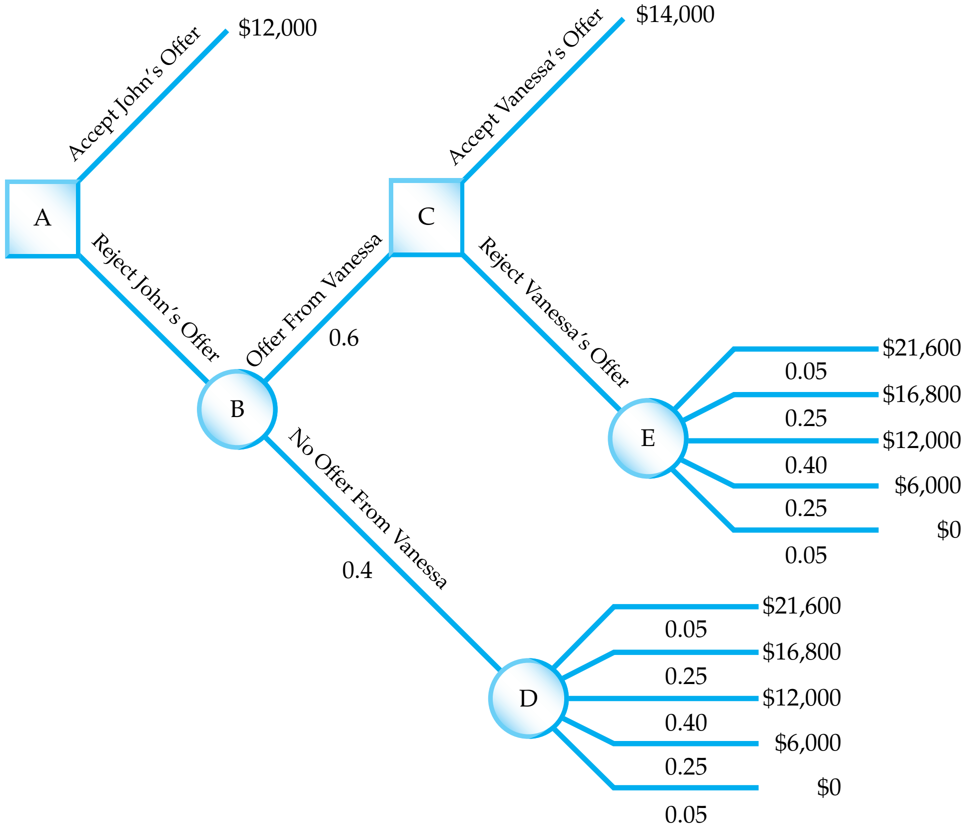


FIGURE 1.6
The completed decision tree.

Fundamental Aspects of Decision Trees

Let us pause and look again at the decision tree as shown in Figure 1.6. Notice that time in the decision tree flows from left to right, and the placement of the decision nodes and the event nodes is logically consistent with the way events will play out in reality. Any event or decision that must logically precede certain other events and decisions is appropriately placed in the tree to reflect this logical dependence.

The tree has two decision nodes, namely node A and node C. Node A represents the decision Bill must make soon: whether to accept or reject John's offer. Node C represents the decision Bill might have to make in late November: whether to accept or reject Vanessa's offer. The branches emanating from each decision node represent all of the possible decisions under consideration at that point in time under the appropriate circumstances.

There are three event nodes in the tree, namely nodes B, D, and E. Node B represents the uncertain event of whether or not Bill will receive a job offer from Vanessa's firm. Node D (and also Node E) represents the uncertain events governing the school's corporate summer recruiting salaries. The branches emanating from each event node represent a set of mutually exclusive and collectively exhaustive outcomes from the event node. Furthermore, the sum of the probabilities of each outcome branch emanating from a given event node must sum to one. (This is because the set of possible outcomes is collectively exhaustive.)

These important characteristics of a decision tree are summarized as follows:

Key Characteristics of a Decision Tree

1. Time in a decision tree flows from left to right, and the placement of the decision nodes and the event nodes is logically consistent with the way events will play out in reality. Any event or decision that must logically precede certain other events and decisions is appropriately placed in the tree to reflect this logical dependence.
2. The branches emanating from each decision node represent all of the possible decisions under consideration at that point in time under the appropriate circumstances.
3. The branches emanating from each event node represent a set of mutually exclusive and collectively exhaustive outcomes of the event node.
4. The sum of the probabilities of each outcome branch emanating from a given event node must sum to one.
5. Each and every "final" branch of the decision tree has a numerical value associated with it. This numerical value usually represents some measure of monetary value, such as salary, revenue, cost, etc.

Notice that in the case of Bill's summer job decision, all of the numerical values associated with the final branches in the decision tree are dollar figures of salaries, which are inherently objective measures to work with. However, Bill might also wish to consider subjective measures in making his decision. We have conveniently assumed for simplicity that the intangible benefits of his summer job options, such as opportunities to learn, networking, resumé-building, etc., would be the same at either his former employer, Vanessa's firm, or in any job offer he might receive through the school's corporate summer recruiting. In reality, these subjective measures would not be the same for all of Bill's possible options. Of course, another important subjective factor, which Bill might also consider, is the value of the time he would have to spend in corporate summer recruiting. Although we will analyze the decision tree ignoring all of these subjective measures, the value of Bill's time should at least be considered when reviewing the conclusions afterward.

Solution of Bill's Problem by Folding Back the Decision Tree

If Bill's choice were simply between accepting a job offer of \$12,000 or accepting a different job offer of \$14,000, then his decision would be easy: he would take the higher salary offer. However, in the presence of uncertainty, it is not necessarily obvious how Bill might proceed.

Suppose, for example, that Bill were to reject John's offer, and that in mid-November he were to receive an offer of \$14,000 from Vanessa's firm. He would then be at node C of the decision tree. How would he go about deciding between obtaining a summer salary of \$14,000 with certainty, and the distribution of possible salaries he might obtain (with varying degrees of uncertainty) from participating in the school's corporate summer recruiting? The criterion that most decision-makers feel is most appropriate to use in this setting is to convert the distribution of possible salaries to a single numerical value using the **expected monetary value** (EMV) of the possible outcomes:

The **expected monetary value** or **EMV** of an uncertain event is the weighted average of all possible numerical outcomes, with the probabilities of each of the possible outcomes used as the weights.

Therefore, for example, the EMV of participating in corporate summer recruiting is computed as follows:

$$\text{EMV} =$$
$$0.05 \times \$21{,}600 + 0.25 \times \$16{,}800 + 0.40 \times \$12{,}000 + 0.25 \times \$6{,}000 + 0.05 \times \$0$$
$$= \$11{,}580.$$

The EMV of a certain event is defined to be the monetary value of the event. For example, suppose that Bill were to receive a job offer from Vanessa's firm, and that he were to accept the job offer. Then the EMV of this choice would simply be $14,000.

Notice that the EMV of the choice to participate in corporate recruiting is $11,580, which is less than $14,000 (the EMV of accepting the offer from Vanessa's firm), and so under the EMV criterion, Bill would prefer the job offer from Vanessa's firm to the option of participating in corporate summer recruiting.

The EMV is one way to convert a group of possible outcomes with monetary values and probabilities to a single number that weighs each possible outcome by its probability. The EMV represents an "averaging" approach to uncertainty. It is quite intuitive, and is quite appropriate for a wide variety of decision problems under uncertainty. (However, there are cases where it is not necessarily the best method for converting a group of possible outcomes to a single number. In Section 1.5, we discuss several aspects of the EMV criterion further.)

Using the EMV criterion, we can now "solve" the decision tree. We do so by evaluating every event node using the EMV of the event node, and evaluating every decision node by choosing that decision which has the best EMV. This is accomplished by starting at the final branches of the tree, and then working "backwards" to the starting node of the decision tree. For this reason, the process of solving the decision tree is called **folding back the decision tree.** It is also occasionally referred to as **backwards induction.** This process is illustrated in the following discussion.

Starting from any one of the "final" nodes of the decision tree, we proceed backwards. As we have already seen, the EMV of node E is $11,580. It is customary to write the EMV of an event node above the node, as is shown in Figure 1.7. Similarly, the EMV of node D is also $11,580, which we write above node D. This is also displayed in Figure 1.7.

We next examine decision node C, which corresponds to the event that Bill receives a job offer from Vanessa's firm. At this decision node, there are two choices. The first choice is for Bill to accept the offer from Vanessa's firm, which has an EMV of $14,000. The second choice is to reject the offer, and instead to participate in corporate summer recruiting, which has an EMV of $11,580. As the EMV of $11,580 is less than the EMV of $14,000, it is better to choose the branch corresponding to accepting Vanessa's offer. Pictorially, we show this by crossing off the inferior choice by drawing two lines through the branch, and by writing the monetary value of the best choice above the decision node. This is shown in Figure 1.7 as well.

We continue by evaluating event node B, which is the event node corresponding to the event where Vanessa's firm either will or will not offer Bill a summer job. The methodology we use is the same as evaluating the salary distributions from participating in corporate summer recruiting. We compute the EMV of the node by computing the

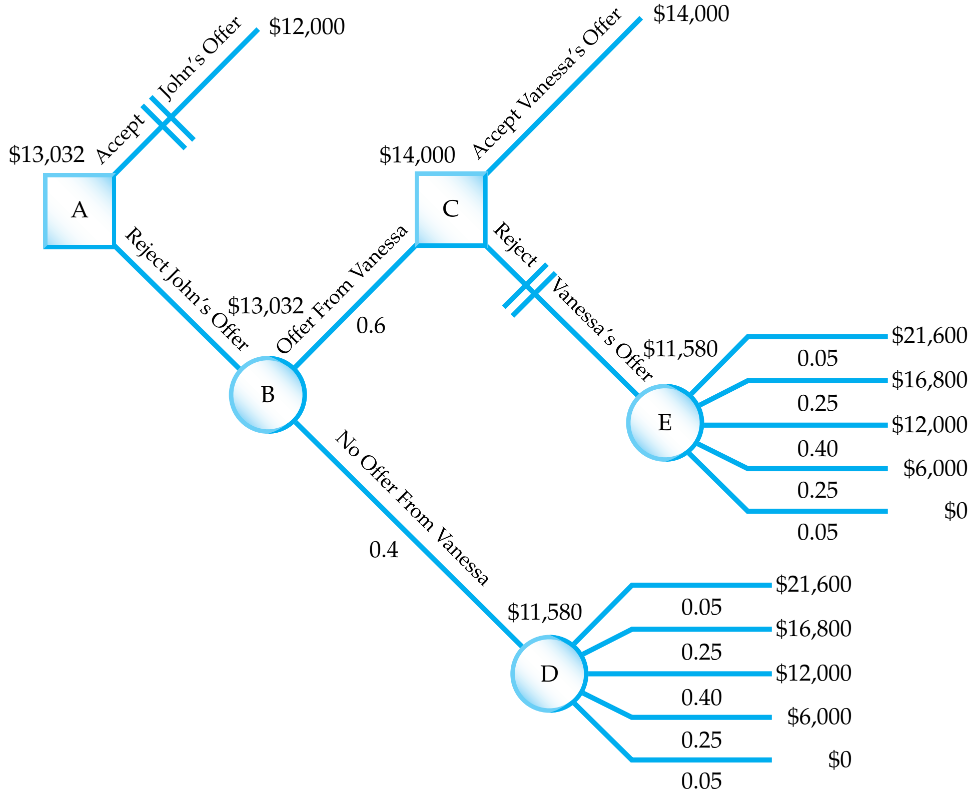


FIGURE 1.7
Solution of the decision tree.

weighted average of the EMVs of each of the outcomes, weighted by the probabilities corresponding to each of the outcomes. In this case, this means multiplying the probability of an offer (0.60) by the $14,000 value of decision node C, then multiplying the probability of not receiving an offer from Vanessa's firm (0.40) times the EMV of node D, which is $11,580, and then adding the two quantities. The calculations are:

$$\text{EMV} = 0.60 \times \$14{,}000 + 0.40 \times \$11{,}580 = \$13{,}032.$$

This number is then placed above the node, as shown in Figure 1.7.

The last step in solving the decision tree is to evaluate the remaining node, which is the first node of the tree. This is a decision node, and its evaluation is accomplished by comparing the better of the two EMV values of the branches that emanate from it. The upper branch, which corresponds to accepting John's offer, has an EMV of $12,000. The lower branch, which corresponds to rejecting John's offer and proceeding onward, has an EMV of $13,032. As this latter value is the highest, we cross off the branch corresponding to accepting John's offer, and place the EMV value of $13,032 above the initial node. The completed solution of the decision tree is shown in Figure 1.7.

Let us now look again at the solved decision tree and examine the "optimal decision strategy" under uncertainty. According to the solved tree, Bill should not accept John's job offer, i.e., he should reject John's job offer. This is shown at the first decision node. Then, if Bill receives a job offer from Vanessa's firm, he should accept this offer. This is shown at the second decision node. Of course, if he does not receive a job offer from Vanessa's firm, he would then participate in the school's corporate summer recruiting program. The EMV of John's optimal decision strategy is $13,032.

Summarizing, Bill's optimal decision strategy can be stated as follows:

Bill's Optimal Decision Strategy:

- Bill should reject John's offer in October.
- If Vanessa's firm offers him a job, he should accept it. If Vanessa's firm does not offer him a summer job, he should participate in the school's corporate summer recruiting.
- The EMV of this strategy is $13,032.

Note that the output from constructing and solving the decision tree is a very concrete plan of action, which states what decisions should be made under each possible uncertain outcome that might prevail.

The procedure for solving a decision tree can be formally stated as follows:

Procedure for Solving a Decision Tree

1. Start with the final branches of the decision tree, and evaluate each event node and each decision node, as follows:
   - For an event node, compute the EMV of the node by computing the weighted average of the EMV of each branch weighted by its probability. Write this EMV number above the event node.
   - For a decision node, compute the EMV of the node by choosing that branch emanating from the node with the best EMV value. Write this EMV number above the decision node, and cross off those branches emanating from the node with inferior EMV values by drawing a double line through them.
2. The decision tree is solved when all nodes have been evaluated.
3. The EMV of the optimal decision strategy is the EMV computed for the starting branch of the tree.

As we mentioned already, the process of solving the decision tree in this manner is called **folding back the decision tree.** It is also sometimes referred to as **backwards induction.**

Sensitivity Analysis of the Optimal Decision

If this were an actual business decision, it would be naive to adopt the optimal decision strategy derived above, without a critical evaluation of the impact of the key data assumptions that were made in the development of the decision tree model. For example, consider the following data-related issues that we might want to address:

- **Issue 1: The probability that Vanessa's firm would offer Bill a summer job.** We have subjectively assumed that the probability that Vanessa's firm would offer Bill a summer job to be 0.60. It would be wise to test how changes in this probability might affect the optimal decision strategy.
- **Issue 2: The cost of Bill's time and effort in participating in the school's corporate summer recruiting.** We have implicitly assumed that the cost of Bill's time and effort in participating in the school's corporate summer recruiting would be zero. It would be wise to test how high the implicit cost of participating

in corporate summer recruiting would have to be before the optimal decision strategy would change.

- **Issue 3: The distribution of summer salaries that Bill could expect to receive.** We have assumed that the distribution of summer salaries that Bill could expect to receive is given by the numbers in Table 1.1. It would be wise to test how changes in this distribution of salaries might affect the optimal decision strategy.

The process of testing and evaluating how the solution to a decision tree behaves in the presence of changes in the data is referred to as **sensitivity analysis.** The process of performing sensitivity analysis is as much an art as it is a science. It usually involves choosing several key data values and then testing how the solution of the decision tree model changes as each of these data values are modified, one at a time. Such a process is very important for understanding what data are driving the optimal decision strategy and how the decision tree model behaves under changes in key data values. The exercise of performing sensitivity analysis is important in order to gain confidence in the validity of the model and is necessary before one bases one's decisions on the output from a decision tree model. We illustrate next the art of sensitivity analysis by performing the three data changes suggested previously.

Note that in order to evaluate how the optimal decision strategy behaves as a function of changes in the data assumptions, we will have to solve and re-solve the decision tree model many times, each time with slightly different values of certain data. Obviously, one way to do this would be to re-draw the tree each time and perform all of the necessary arithmetic computations by hand each time. This approach is obviously very tedious and repetitive, and in fact we can do this much more conveniently with the help of a computer spreadsheet. We can represent the decision tree problem and its solution very conveniently on a spreadsheet, illustrated in Figure 1.8 and explained in the following discussion.

FIGURE 1.8
Spreadsheet representation of Bill Sampras' summer job problem.

	Spreadsheet Representation of Bill Sampras' Decision Problem		
Data			
Value of John's offer	$12,000		
Value of Vanessa's offer	$14,000		
Probability of offer from Vanessa's firm	0.60		
Cost of participating in Recruiting	$0		
	Distribution of Salaries from Recruiting		
	Weekly Salary	**Total Summer Pay**	**Percentage of Students**
		(based on 12 weeks)	**who Received this Salary**
	$1,800	$21,600	5%
	$1,400	$16,800	25%
	$1,000	$12,000	40%
	$500	$6,000	25%
	$0	$0	5%
	EMV of Nodes		
	Nodes	**EMV**	
	A	$13,032	
	B	$13,032	
	C	$14,000	
	D	$11,580	
	E	$11,580	

In the spreadsheet representation of Figure 1.8, the data for the decision tree is given in the upper part of the spreadsheet, and the "solution" of the spreadsheet is computed in the lower part in the "EMV of Nodes" table. The computation of the EMV of each node is performed automatically as a function of the data. For example, we know that node E of the spreadsheet has its EMV computed as follows:

$$\text{EMV of node E} =$$
$$0.05 \times \$21{,}600 + 0.25 \times \$16{,}800 + 0.40 \times \$12{,}000 + 0.25 \times \$6{,}000 + 0.05 \times \$0$$
$$= \$11{,}580.$$

The EMV of node D is computed in an identical manner. As presented earlier, the EMV of node C is the maximum of the EMV of node E and the value of an offer from Vanessa's firm, and is computed as

$$\text{EMV of node C} = \text{MAX}\{\text{EMV of node E}, \$14{,}000\}.$$

Similarly, the EMV of nodes B and A are given by

$$\text{EMV of node B} = (0.60) \times (\text{EMV of node C}) + (1 - 0.60) \times (\text{EMV of node D})$$

and

$$\text{EMV of node A} = \text{MAX}\{\text{EMV of node B}, \$12{,}000\}.$$

All of these formulas can be conveniently represented in a spreadsheet, and such a spreadsheet is shown in Figure 1.8. Note that the EMV numbers for all of the nodes in the spreadsheet correspond exactly to those computed "by hand" in the solution of the decision tree shown in Figure 1.7.

We now show how the spreadsheet representation of the decision tree can be used to study how the optimal decision strategy changes relative to the three key data issues discussed above at the start of this subsection. To begin, consider the first issue, which concerns the sensitivity of the optimal decision strategy to the value of the probability that Vanessa's firm will offer Bill a summer job. Denote this probability by p, i.e.,

$$p = \text{probability that Vanessa's firm will offer Bill a summer job.}$$

If we test a variety of values of p in the spreadsheet representation of the decision tree, we will find that the optimal decision strategy (which is to reject John's job offer, and to accept a job offer from Vanessa's firm if it is offered) remains the same for all values of p greater than or equal to $p = 0.174$. Figure 1.9 shows the output of the spreadsheet when $p = 0.18$, for example, and notice that the EMV of node B is \$12,016, which is just barely above the threshold value of \$12,000. For values of p at or below $p = 0.17$, the EMV of node B becomes less than \$12,000, which results in a new optimal decision strategy of accepting John's job offer. We can conclude the following:

- As long as the probability of Vanessa's firm offering Bill a job is 0.18 or larger, then the optimal decision strategy will still be to reject John's offer and to accept a summer job with Vanessa's firm if they offer it to him.

This is reassuring, as it is reasonable for Bill to be very confident that the probability of Vanessa's firm offering him a summer job is surely greater than 0.18.

We next use the spreadsheet representation of the decision tree to study the second data assumption issue, which concerns the sensitivity of the optimal decision strategy to the implicit cost to Bill (in terms of his time) of participating in the school's corporate summer recruiting program. Denote this cost by c, i.e.,

$$c = \text{implicit cost to Bill of participating in the school's corporate summer recruiting program.}$$

FIGURE 1.9
Output of the spreadsheet of Bill Sampras' summer job problem when the probability that Vanessa's firm will make Bill an offer is 0.18.

	Spreadsheet Representation of Bill Sampras' Decision Problem		
Data			
Value of John's offer	$12,000		
Value of Vanessa's offer	$14,000		
Probability of offer from Vanessa's firm	0.18		
Cost of participating in Recruiting	$0		
	Distribution of Salaries from Recruiting		
	Weekly Salary	**Total Summer Pay**	**Percentage of Students**
		(based on 12 weeks)	**who Received this Salary**
	$1,800	$21,600	5%
	$1,400	$16,800	25%
	$1,000	$12,000	40%
	$500	$6,000	25%
	$0	$0	5%
	EMV of Nodes		
	Nodes	**EMV**	
	A	$12,016	
	B	$12,016	
	C	$14,000	
	D	$11,580	
	E	$11,580	

If we test a variety of values of c in the spreadsheet representation of the decision tree, we will notice that the current optimal decision strategy (which is to reject John's job offer, and to accept a job offer from Vanessa's firm if it is offered) remains the same for all values of c less than $c = \$2{,}578$. Figure 1.10 shows the output of the spreadsheet when $c = \$2{,}578$. For values of c above $c = \$2{,}578$, the EMV of node B becomes less than $12,000, which results in a new optimal decision strategy of accepting John's job offer. We can conclude the following:

- As long as the implicit cost to Bill of participating in summer recruiting is less than $2,578, then the optimal decision strategy will still be to reject John's offer and to accept a summer job with Vanessa's firm if they offer it to him.

This is also reassuring, as it is reasonable to estimate that the implicit cost to Bill of participating in the school's corporate summer recruiting program is much less than $2,578.

We next use the spreadsheet representation of the decision tree to study the third data issue, which concerns the sensitivity of the optimal decision strategy to the distribution of possible summer job salaries from participating in corporate recruiting. Recall that Table 1.1 contains the data for the salaries Bill might possibly realize by participating in corporate summer recruiting. Let us explore the consequences of changing all of the possible salary offers of Table 1.1 by an amount S. That is, we will explore modifying Bill's possible summer salaries by an amount S. If we test a variety of values of S in the spreadsheet representation of the model, we will notice that the current optimal decision strategy remains optimal for all values of S less than $S = \$2{,}419$. Figure 1.11 shows the output of the spreadsheet when $S = \$2{,}419$. For values of S above $S = \$2{,}420$, the EMV of node E will become greater than or equal to $14,000, and consequently Bill's optimal decision strategy will change: he would reject an offer from Vanessa's firm if it materialized, and instead would participate in the school's corporate summer recruiting program. We can conclude:

FIGURE 1.10
Output of the spreadsheet of Bill Sampras' summer job problem if the cost of Bill's time spent participating in corporate summer recruiting is $2,578.

	Spreadsheet Representation of Bill Sampras' Decision Problem		
Data			
Value of John's offer	$12,000		
Value of Vanessa's offer	$14,000		
Probability of offer from Vanessa's firm	0.60		
Cost of participating in Recruiting	$2,578		
	Distribution of Salaries from Recruiting		
	Weekly Salary	**Total Summer Pay**	**Percentage of Students**
		(based on 12 weeks)	**who Received this Salary**
	$1,800	$21,600	5%
	$1,400	$16,800	25%
	$1,000	$12,000	40%
	$500	$6,000	25%
	$0	$0	5%
	EMV of Nodes		
	Nodes	**EMV**	
	A	$12,001	
	B	$12,001	
	C	$14,000	
	D	$9,002	
	E	$9,002	

FIGURE 1.11
Output of the spreadsheet of Bill Sampras' summer job problem if summer salaries from recruiting were $2,419 higher.

	Spreadsheet Representation of Bill Sampras' Decision Problem		
Data			
Value of John's offer	$12,000		
Value of Vanessa's offer	$14,000		
Probability of offer from Vanessa's firm	0.60		
Cost of participating in Recruiting	$0		
	Distribution of Salaries from Recruiting		
	Weekly Salary	**Total Summer Pay**	**Percentage of Students**
		(based on 12 weeks)	**who Received this Salary**
	$1,800	$24,019	5%
	$1,400	$19,219	25%
	$1,000	$14,419	40%
	$500	$8,419	25%
	$0	$2,419	5%
	EMV of Nodes		
	Nodes	**EMV**	
	A	$14,000	
	B	$14,000	
	C	$14,000	
	D	$13,999	
	E	$13,999	

- In order for Bill's optimal decision strategy to change, all of the possible summer corporate recruiting salaries of Table 1.1 would have to increase by more than $2,419.

This is also reassuring, as it is reasonable to anticipate that summer salaries from corporate summer recruiting in general would not be $2,419 higher this coming summer than they were last summer.

We can summarize our findings as follows:

- For all three of the data issues that we have explored (the probability p of Vanessa's firm offering Bill a summer job, the implicit cost c of participating in corporate summer recruiting, and an increase S in all possible salary values from corporate summer recruiting), we have found that the optimal decision strategy does not change unless these quantities take on unreasonable values. Therefore, it is safe to proceed with confidence in recommending to Bill Sampras that he adopt the optimal decision strategy found in the solution to the decision tree model. Namely, he should reject John's job offer, and he should accept a job offer from Vanessa's firm if such an offer is made.

In some applications of decision analysis, the decision-maker might discover that the optimal decision strategy is very sensitive to a key data value. If this happens, it is then obviously important to spend some effort to determine the most reasonable value of that data. For instance, in the decision tree we have constructed, suppose that in fact the optimal decision was very sensitive to the probability p that Vanessa's firm would offer Bill a summer job. We might then want to gather data on how many offers Vanessa's firm made to Sloan students in previous years, and in particular we might want to look at how students with Bill's general profile fared when they applied for jobs with Vanessa's firm. This information could then be used to develop a more exact estimate of the probability p that Bill would receive a job offer from Vanessa's firm.

Note that in this sensitivity analysis exercise, we have only changed one data value at a time. In some problem instances, the decision-maker might want to test how the model behaves under simultaneous changes in more than one data value. This is a bit more difficult to analyze, of course.

1.2 SUMMARY OF THE GENERAL METHOD OF DECISION ANALYSIS

The example of Bill Sampras' summer job decision problem illustrates the format of the general method of **decision analysis** to systematically analyze a decision problem. The format of this general method is as follows:

Principal Steps of Decision Analysis

1. Structure the decision problem. List all of the decisions that have to be made. List all of the uncertain events in the problem and all of their possible outcomes.

2. Construct the basic decision tree by placing the decision nodes and the event nodes in their chronological and logically consistent order.

3. Determine the probability of each of the possible outcomes of each of the uncertain events. Write these probabilities on the decision tree.

4. Determine the numerical values of each of the final branches of the decision tree. Write these numerical values on the decision tree.

5. Solve the decision tree using the folding-back procedure:

(a) Start with the final branches of the decision tree, and evaluate each event node and each decision node, as follows:

- For an event node, compute the EMV of the node by computing the weighted average of the EMV of each branch weighted by its probability. Write this EMV number above the event node.
- For a decision node, compute the EMV of the node by choosing that branch emanating from the node with the best EMV value. Write this EMV number above the decision node and cross off those branches emanating from the node with inferior EMV values by drawing a double line through them.

(b) The decision tree is solved when all nodes have been evaluated.

(c) The EMV of the optimal decision strategy is the EMV computed for the starting branch of the tree.

6. Perform sensitivity analysis on all key data values. For each data value for which the decision-maker lacks confidence, test how the optimal decision strategy will change relative to a change in the data value, one data value at a time.

As mentioned earlier, the solution of the decision tree and the sensitivity analysis procedure typically involve a number of mechanical arithmetic calculations. Unless the decision tree is small, it might be wise to construct a spreadsheet version of the decision tree in order to perform these calculations automatically and quickly. (And of course, a spreadsheet version of the model will also eliminate the likelihood of making arithmetical errors!)

1.3 ANOTHER DECISION TREE MODEL AND ITS ANALYSIS

In this section, we continue to illustrate the methodology of decision analysis by considering a strategic development decision problem encountered by a new company called Bio-Imaging, Incorporated.

BIO-IMAGING DEVELOPMENT STRATEGIES

In 2004, the company Bio-Imaging, Incorporated was formed by James Bates, Scott Tillman, and Michael Ford, in order to develop, produce, and market a new and potentially extremely beneficial tool in medical diagnosis. Scott Tillman and James Bates were each recent graduates from Massachusetts Institute of Technology (MIT), and Michael Ford was a professor of neurology at Massachusetts General Hospital (MGH). As part of his graduate studies at MIT, Scott had developed a new technique and a software package to process MRI (magnetic resonance imaging) scans of brains of patients using a personal computer. The software, using state of the art computer graphics, would construct a three-dimensional picture of a patient's brain and could be used to find the exact location of a brain lesion or a brain tumor, estimate its volume and shape, and even locate the centers in the brain that would be affected by the tumor. Scott's work was an extension of earlier two-dimensional image processing work developed by James, which had been used extensively in Michael Ford's medical group at MGH for analyzing the effects of lesions on patients' speech difficulties. Over the last few years, this software program had been used to make relatively accurate measurements and diagnoses of brain lesions and tumors.

Although not yet fully tested, Scott's more advanced three-dimensional software program promised to be much more accurate than other methods in diagnosing lesions. While a variety of other scientists around the world had developed their own MRI imaging software, Scott's new three-dimensional program was very different and far superior to any other existing software for MRI image processing.

At James' recommendation, the three gentlemen formed Bio-Imaging, Incorporated with the goal of developing and producing a commercial software package that hospitals and doctors could use. Shortly thereafter, they were approached by the Medtech Corporation, a large medical imaging and software company. Medtech offered them $150,000 to buy the software package in its then-current form, together with the rights to develop and market the software world-wide. The other two partners authorized James (who was the "businessman" of the partnership) to decide whether or not to accept the Medtech offer. If they rejected the offer, their plan was to continue their own development of the software package in the next six months. This would entail an investment of approximately $200,000, which James felt could be financed through the partners' personal savings.

If Bio-Imaging were successful in their effort to make the three-dimensional prototype program fully operational, they would face two alternative development strategies. One alternative would be to apply after six months time for a $300,000 Small Business Innovative Research (SBIR) grant from the National Institutes of Health (NIH). The SBIR money would then be used to further develop and market their product. The other alternative would be to seek further capital for the project from a venture capital firm. In fact, Michael had had several discussions with the venture capital firm Nugrowth Development. Nugrowth Development had proposed that if Bio-Imaging were successful in producing a three-dimensional prototype program, Nugrowth would then offer $1,000,000 to Bio-Imaging to finance and market the software package in exchange for 80% of future profits after the three-dimensional prototype program became fully operational. (Because NIH regulations do not allow a company to receive an NIH grant and also receive money from a venture capital firm, Bio-Imaging would not be able to receive funding from both sources.)

James knew that there was substantial uncertainty concerning the likelihood of receiving the SBIR grant. He also knew that there was substantial uncertainty about how successful Bio-Imaging might be in marketing their product. He felt, however, that if they were to accept the Nugrowth Development offer, the profitability of the product would probably then be higher than if they were to market the product themselves.

If Bio-Imaging was not successful in making the three-dimensional prototype program fully operational, James felt that they could still apply for an SBIR grant with the two-dimensional software program. He realized that in this case, they would be less likely to be awarded the SBIR grant. Furthermore, clinical tests would be needed to fine-tune the two-dimensional program prior to applying for the grant. James estimated that the cost of these additional tests would be around $100,000.

The decision problem faced by Bio-Imaging was whether to accept the offer from Medtech or to continue the research and development of the three-dimensional software package. If they were successful in producing a three-dimensional prototype, they would have to decide either to apply for the SBIR grant or to accept the offer from Nugrowth. If they were not successful in producing a three-dimensional prototype, they would have to decide either to further invest in the two-dimensional product and apply for an SBIR grant, or to abandon the project altogether. In the midst of all of this, James also wondered whether the cost of the Nugrowth offer (80% of future profits) might be too high relative to the benefits ($1,000,000 in much-needed capital). Clearly James needed to think hard about the decisions Bio-Imaging was facing.

Data Estimates of Revenues and Probabilities

Given the intense competition in the market for medical imaging technology, James knew that there was substantial uncertainty surrounding the potential revenues of Bio-Imaging over the next three years. James tried to estimate these revenues under a variety of possible scenarios. Table 1.2 shows James' data estimates of revenues under three scenarios ("high profit," "medium profit," and "low profit") in the event that the three-dimensional prototype were to become operational and if they were to receive the SBIR grant. Under the "high profit" scenario the program would presumably be very successful in the marketplace, yielding total revenues of $3,000,000. In the "medium profit" scenario, James estimated the revenues to be $500,000, while in the "low profit" scenario, he estimated that there would be no revenues. James assigned his estimated probabilities of these three scenarios to be 20%, 40%, and 40% for the "high profit," "medium profit," and "low profit" scenarios, respectively.

Table 1.3 shows James' data estimates of revenues of Bio-Imaging in the event that the three-dimensional prototype were to become operational and if they were to accept the financing offer from Nugrowth Development. Given the higher resources that would be available to them ($1,000,000 of capital), James estimated that under the "high profit" scenario the program would yield total revenues of $10,000,000. In the "medium profit" scenario, James estimated the revenues to be $3,000,000; while in the "low profit" scenario, he estimated that there would be no revenues. As before, James assigned his estimated probabilities of the three scenarios to be 20%, 40%, and 40% for the "high profit," "medium profit," and "low profit" scenarios, respectively.

Table 1.4 shows James' data estimates of revenues of Bio-Imaging in the event that the three-dimensional prototype were not successful and if they were to receive

TABLE 1.2
Estimated revenues of Bio-Imaging, if the three-dimensional prototype were operational and if Bio-Imaging were awarded the SBIR grant.

Scenario	*Probability*	*Total Revenues*
High Profit	20%	$3,000,000
Medium Profit	40%	$500,000
Low Profit	40%	$0

TABLE 1.3
Estimated revenues of Bio-Imaging, if the three-dimensional prototype were operational, under financing from Nugrowth Development.

Scenario	*Probability*	*Total Revenues*
High Profit	20%	$10,000,000
Medium Profit	40%	$3,000,000
Low Profit	40%	$0

TABLE 1.4
Estimated profit of Bio-Imaging, if the three-dimensional prototype were unsuccessful and if Bio-Imaging were awarded the SBIR grant.

Scenario	*Probability*	*Total Revenues*
High Profit	25%	$1,500,000
Low Profit	75%	$0

the SBIR grant for the two-dimensional software program. In this case James considered only two scenarios: "high profit" and "low profit." Note that the revenue estimates are quite low. Under the "high profit" scenario the program would yield total revenues of $1,500,000. In the "low profit" scenario, James estimated that there would be no revenues. James assigned his estimated probabilities of the scenarios to be 25% and 75% for the "high profit" and the "low profit" scenarios, respectively.

James also gave serious thought and analysis to various other uncertainties facing Bio-Imaging. After consulting with Scott, he assigned a 60% likelihood that they would be successful in producing an operational version of the three-dimensional software program. Moreover, after consulting with Michael Ford, James also estimated that the likelihood of winning the SBIR grant after successful completion of the three-dimensional software program to be 70%. However, they estimated that the likelihood of winning the SBIR grant with only the two-dimensional software program would be only 20%.

Construction of the Decision Tree

Let us now construct a decision tree for analyzing the decisions faced by Bio-Imaging. The first decision that must be made is whether Bio-Imaging should accept the offer from Medtech or instead continue with the research and development of the three-dimensional software program. This decision is represented by a decision node with two branches emanating from it, as shown in Figure 1.12. (The label "A" is placed on the decision node so that we can conveniently refer to it later.)

If Bio-Imaging were to accept the offer from Medtech, there would be nothing left to decide. If instead they were to continue with the research and development of the three-dimensional software program, they would find out after six months whether or not they would succeed in making it operational. This event is represented by an event node labeled "B" in Figure 1.13.

If Bio-Imaging were successful in developing the program, they would then face the decision whether to apply for an SBIR grant or to instead accept the offer from Nugrowth Development. This decision is represented as node C in Figure 1.14. If they were to apply for an SBIR grant, they would then either win the grant or not. This event is represented as node E in Figure 1.14. If they were to win the grant, they would then complete the development of the three-dimensional software product and market the product accordingly. The revenues that they would then receive would be in accordance with James' estimates given in Table 1.2. The event node G in Figure 1.14 represents James' estimate of the uncertainty regarding these revenues.

If, however, Bio-Imaging were to lose the grant, the offer from Nugrowth would then not be available either, due to the inherent delays in the grant decision process at NIH. In this case, Bio-Imaging would not have the resources to continue, and would have to abandon the project altogether. This possibility is represented in the lower branch emanating from node E in Figure 1.14.

On the other hand, if Bio-Imaging were to accept the offer from Nugrowth Development, they would then face the revenue possibilities in accordance with James' estimates given in Table 1.3. The event node H in Figure 1.14 represents James' estimate of the uncertainty regarding these revenues.

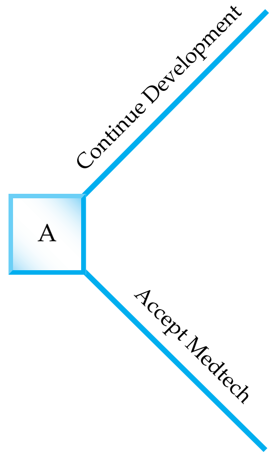


FIGURE 1.12
Representation of the first decision faced by Bio-Imaging.

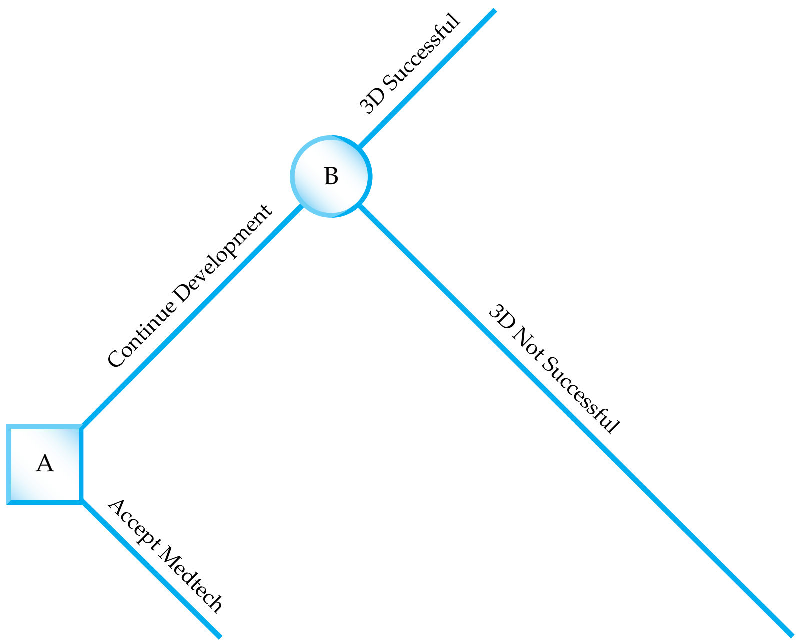


FIGURE 1.13
Further construction of the Bio-Imaging decision tree.

If Bio-Imaging were not successful in making the three-dimensional product operational, there would then be only two alternative choices of action: Abandon the project, or do more work to enhance the two-dimensional software product and then apply for an SBIR grant with the two-dimensional product. These two alternatives are represented at decision node D in Figure 1.15. If Bio-Imaging were to apply for the SBIR grant in this case, they would either win the grant or not. This event is represented as event node F in Figure 1.15. If they were to apply for the SBIR grant and were to win it, they would then complete the development of the two-dimensional software product and market it accordingly. The revenues they would then receive would be in accordance with James' estimates given in Table 1.4. The event node I in Figure 1.15 represents James' estimate of the uncertainty regarding these revenues. Finally, if they were to lose the SBIR grant for the two-dimensional product, they would have to abandon the project, as represented by the lower branch emanating from node F of Figure 1.15.

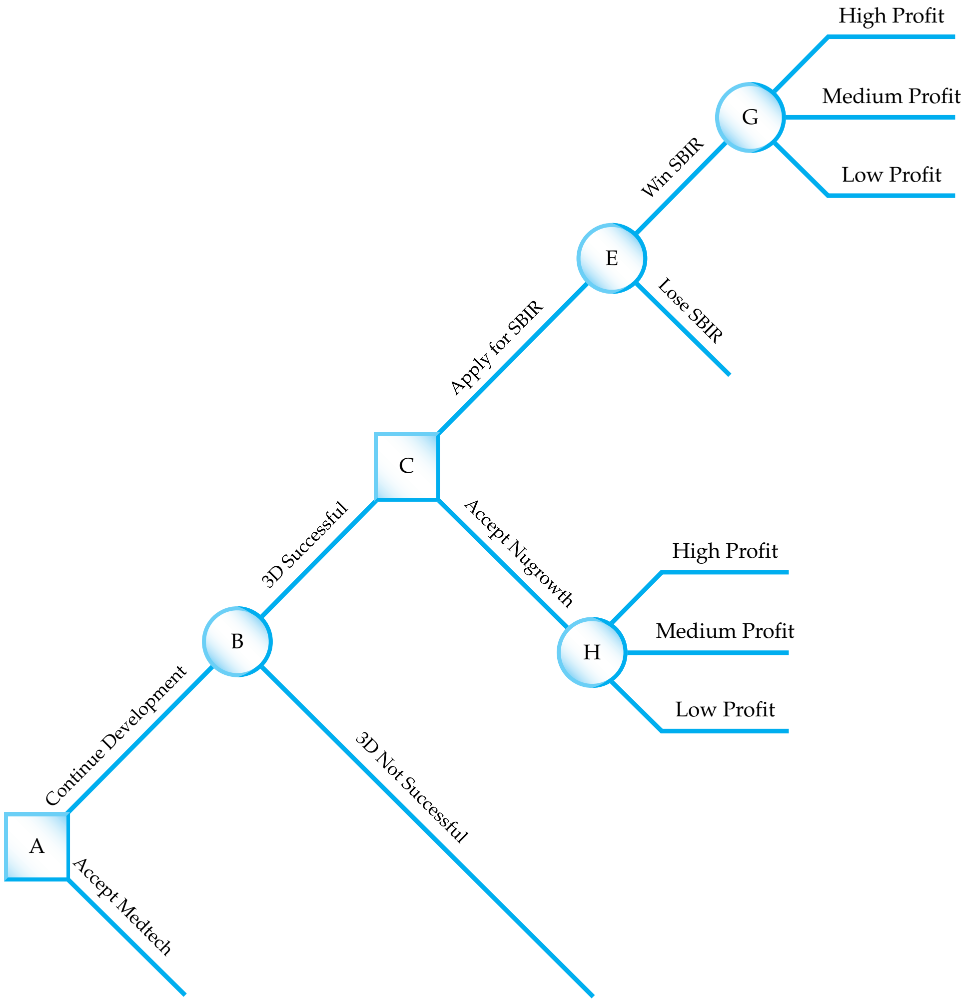


FIGURE 1.14
Further construction of the Bio-Imaging decision tree.

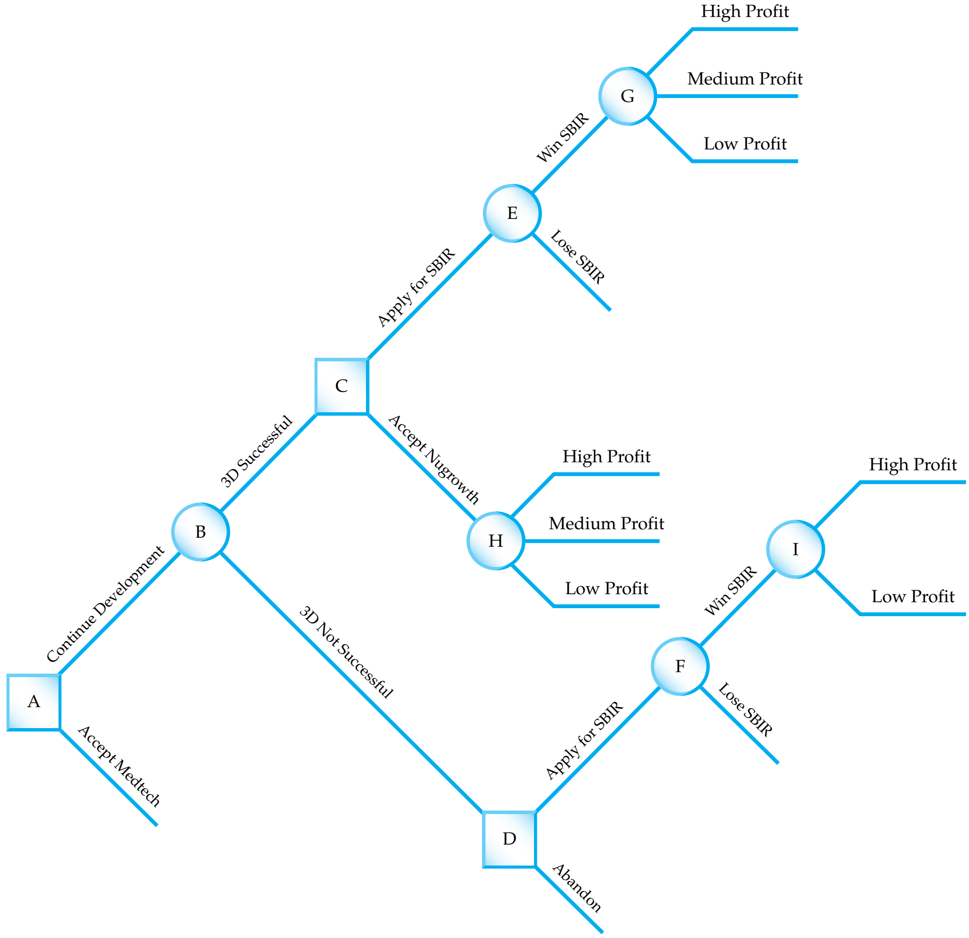
At this point, we have represented a description of the decision problem faced by Bio-Imaging in the decision tree of Figure 1.15. Notice that Figure 1.15 represents all of the decisions and all of the relevant uncertainties in the problem, and portrays the logical unfolding of decisions and uncertain events that Bio-Imaging is facing.

Assigning Probabilities in the Decision Tree

The next task in the decision analysis process is to assign probabilities to all of the uncertain events. For the problem at hand, this task is quite straightforward. For event node B, we place the probability that Bio-Imaging will succeed in producing an operational version of the three-dimensional software under the upper branch emanating from the node. This probability was estimated by James to be 0.60, and so we place this probability under the branch as shown in Figure 1.16. The probability that Bio-Imaging would not be successful in producing a three-dimensional prototype software program is therefore 0.40, and so we place this probability under the lower branch emanating from node B.

For event node E, we place the probability that Bio-Imaging would win the SBIR grant (if they were to succeed at producing an operational version of the three-dimensional software), which James had estimated to be 0.70, under the upper branch emanating from node E. We place the probability of 0.30, which is the proba-

FIGURE 1.15
Representation of the Bio-Imaging decision tree.



bility that Bio-Imaging would not win the SBIR grant under this scenario, under the lower branch emanating from node E.

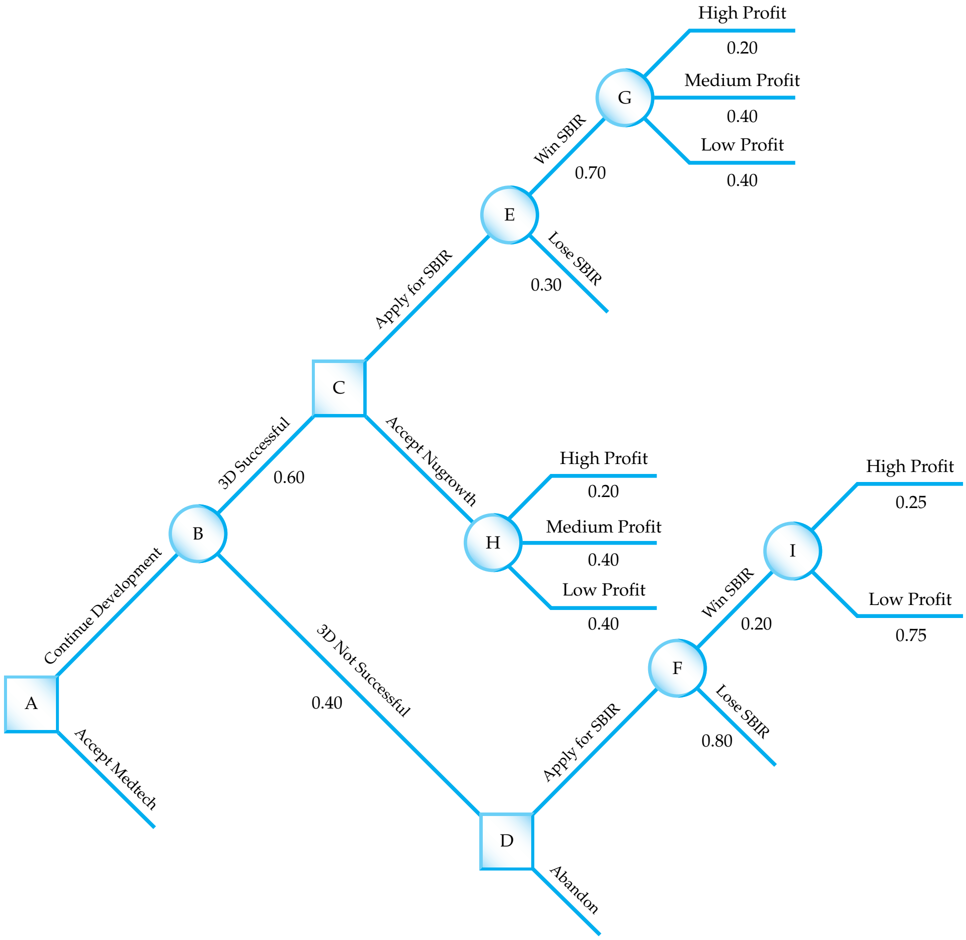
For event node F, we have a similar situation. We place the probability that Bio-Imaging would win the SBIR grant (if they were not to succeed at producing an operational version of the three-dimensional software), which James had estimated to be 0.20, under the upper branch emanating from node F. We place the probability of 0.80, which is the probability that Bio-Imaging would not win the SBIR grant under this scenario, under the lower branch emanating from node F.

Finally, we assign the various probabilities of high, medium, and/or low profit scenarios to the various branches emanating from nodes G, H, and I, according to James' estimates of these scenarios as given in Table 1.2, Table 1.3, and Table 1.4.

Valuing the Final Branches

The remaining step in constructing the decision tree is to assign numerical values to the final branches of the tree. For the Bio-Imaging decision problem, this means computing the net profit corresponding to each final branch of the tree. For the branch corresponding to accepting the offer from Medtech, the computation is trivial: Bio-Imaging would receive $150,000 in net profit if they were to accept the offer from Medtech. This is shown in the lower branch emanating from node A, in Figure 1.17. For all other final branches of the tree, the computation is not quite so easy.

FIGURE 1.16
Representation of the probabilities of all of the uncertain events in the Bio-Imaging decision tree.



Let us consider first the branches emanating from node G. The net profit is the total revenues minus the relevant costs. For these branches, the total revenues are given in Table 1.2. The relevant costs are the costs of research and development of the operational three-dimensional software (which was estimated by James to be \$200,000). The SBIR grant money would be used to cover all final development and marketing costs, and so would not figure into the net profit computation. Therefore, by way of example, the net profit under the "high profit" scenario branch emanating from node G is computed as:

$$\$2{,}800{,}000 = \$3{,}000{,}000 - \$200{,}000.$$

The other two net profit computations for the branches emanating from node G are computed in a similar manner. These numbers are then placed next to their respective branches, as shown in Figure 1.17.

Next consider the lower branch emanating from node E, which corresponds to succeeding at making an operational version of the three-dimensional software, applying for the SBIR grant, and losing the competition for the grant. In this case, the revenues would be zero, as Bio-Imaging would abandon the project, but the costs would still be the \$200,000 in development costs for the operational version of the three-dimensional software. Therefore the net profit would be −\$200,000, as shown in Figure 1.17.

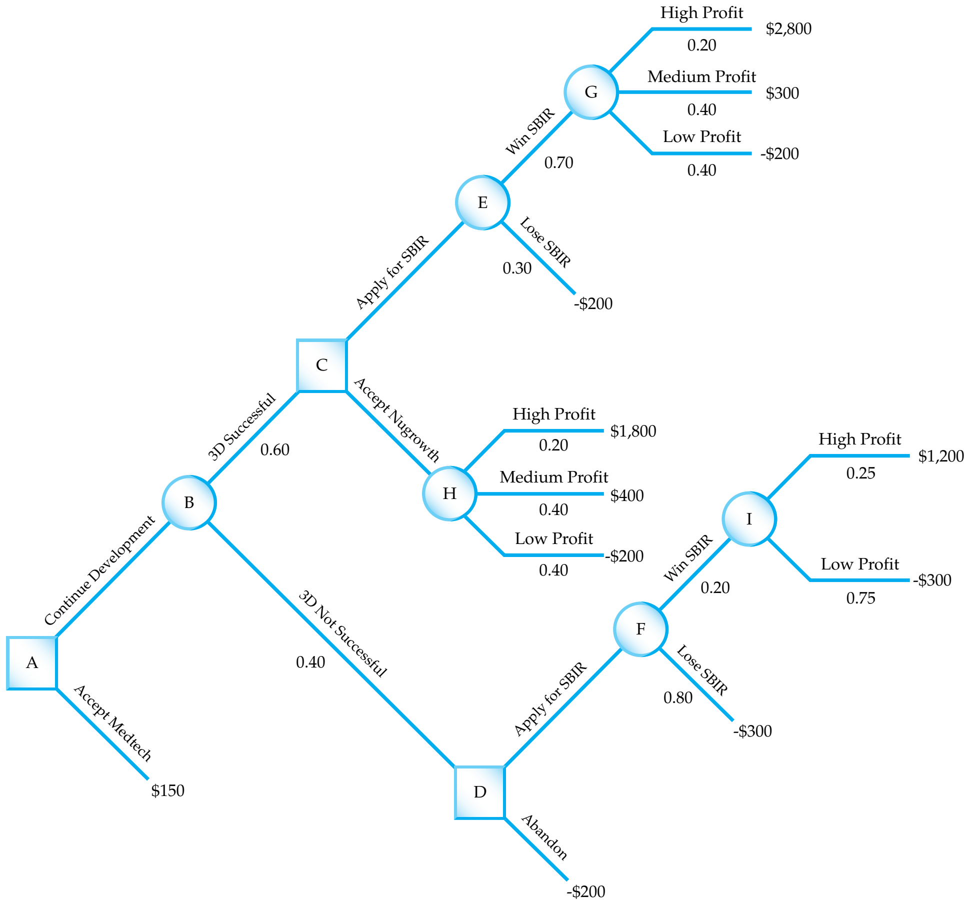


FIGURE 1.17
The complete decision tree for Bio-Imaging (all dollar values are in $1,000).

Next consider the three final branches emanating from node H. At node H, Bio-Imaging would have decided to accept the offer from Nugrowth Development, and so Bio-Imaging would only receive 20% of the total revenues. Under the "high profit" scenario, for example, the total revenues would be $10,000,000 (see Table 1.3). Therefore, the net profits to Bio-Imaging would be:

$$\$1{,}800{,}000 = 0.20 \times \$10{,}000{,}000 - \$200{,}000.$$

The other two net profit computations for the branches emanating from node H are computed in a similar manner. These numbers are then placed next to their respective branches, as shown in Figure 1.17.

Next consider the two final branches emanating from node I. At node I, Bio-Imaging would have been unsuccessful at producing an operational version of the three-dimensional product (at a cost of $200,000), but would have decided to go ahead and further refine the two-dimensional product (at a cost of $100,000). Under the "high profit" scenario, for example, the total revenues would be $1,500,000, see Table 1.4. Therefore, the net profits to Bio-Imaging would be:

$$\$1{,}200{,}000 = \$1{,}500{,}000 - \$200{,}000 - \$100{,}000.$$

The other net profit computation for the "low profit" scenario branch emanating from node I is computed in a similar manner. These numbers are then placed next to their respective branches, as shown in Figure 1.17.

Finally, we compute the net profits for the lower branches of nodes D and F. Using the logic herein, the net profit for the lower branch of node D is $-\$200{,}000$ and the net profit for the lower branch of node F is $-\$300{,}000$. These numbers are shown in Figure 1.17.

At this point, the description of the decision tree is complete. The next step is to solve the decision tree.

Solving for the Optimal Decision Strategy

We now proceed to solve for the optimal decision strategy by folding back the tree. Recall that this entails computing the EMV (expected monetary value) of all event nodes, and computing the EMV of all decision nodes by choosing that decision with the best EMV at the node. Let us start with node H of the tree. At this node, one of the three scenarios ("high profit," "medium profit" and "low profit") would transpire. We compute the EMV of this node by weighting the three outcomes emanating from the node by their corresponding probabilities. Therefore the EMV of node H is computed as:

$$0.20\ (\$1{,}800{,}000)\ +\ 0.40\ (\$400{,}000)\ +\ 0.40\ (-\$200{,}000)\ =\ \$440{,}000.$$

We then write the EMV number $440,000 above node H, as shown in Figure 1.18.

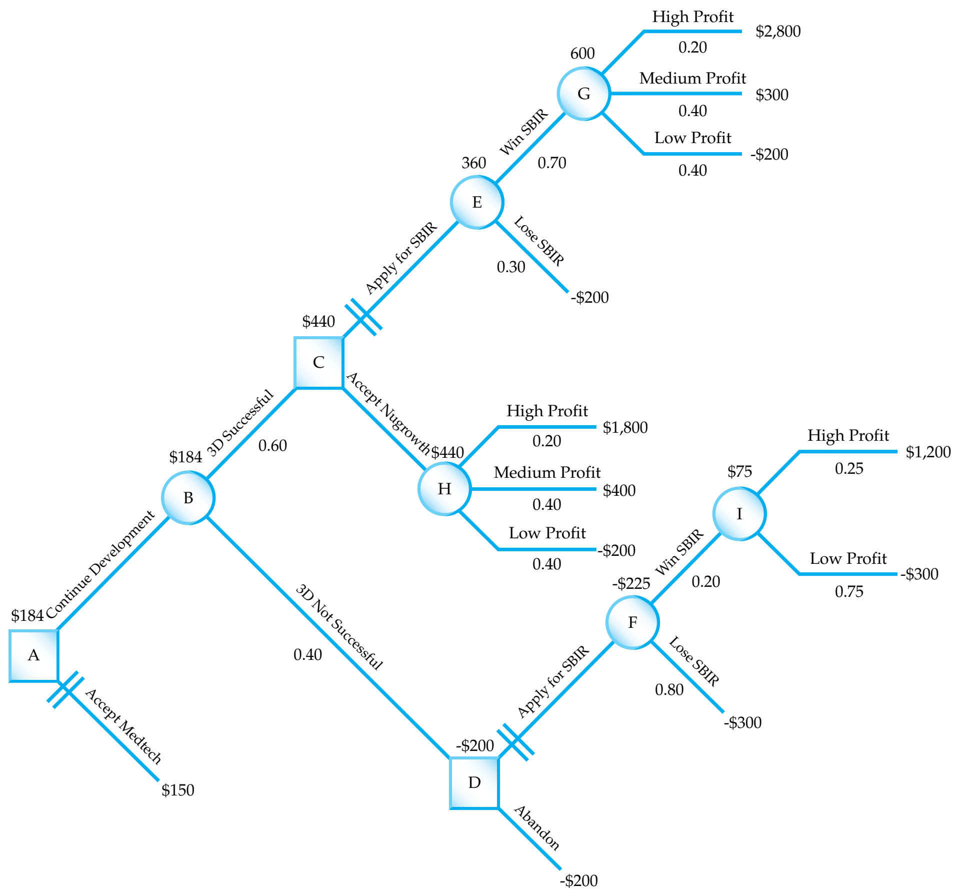


FIGURE 1.18
Solution of the Bio-Imaging decision tree (all dollar values are in $1,000).

Similarly we compute the EMV of node G as follows:

$$0.20\ (\$2{,}800{,}000)\ +\ 0.40\ (\$300{,}000)\ +\ 0.40\ (-\$200{,}000)\ =\ \$600{,}000,$$

and so we write the EMV number \$600,000 above node G, as shown in Figure 1.18.

The EMV of node E is then computed as:

$$0.70\ (\$600{,}000)\ +\ 0.30\ (-\$200{,}000)\ =\ \$360{,}000,$$

and so we write the EMV number \$360,000 above node E, as shown in Figure 1.18.

Node C corresponds to deciding whether to apply for an SBIR grant, or to accept the offer from Nugrowth Development. The EMV of the SBIR option (node E) is \$360,000, while the EMV of the Nugrowth offer is \$440,000. The option with the highest EMV is to accept the offer from the Nugrowth Development. Therefore, we assign the EMV of node C to be the higher of the two EMV values, i.e., \$440,000, and we cross off the branch corresponding to applying for the SBIR grant, as shown in Figure 1.18.

The EMV of node I is computed as:

$$0.25\ (\$1{,}200{,}000)\ +\ 0.75\ (-\$300{,}000)\ =\ \$75{,}000,$$

and so we write the EMV number \$75,000 above node I, as shown in Figure 1.18.

The EMV of node F is

$$0.2\ (\$75{,}000)\ +\ 0.8\ (-\$300{,}000)\ =\ -\$225{,}000,$$

and so we write the EMV number $-$\$225,000 above node F, as shown in Figure 1.18.

At node D, the choice with the highest EMV is to abandon the project, since this alternative has a higher EMV ($-$\$200,000) compared to that of applying for an SBIR grant with the two-dimensional program ($-$\$225,000). Therefore, we assign the EMV of node D to be the higher of the two EMV values, i.e., $-$\$200,000, and we cross off the branch corresponding to applying for the SBIR grant, as shown in Figure 1.18.

The EMV of node B is computed as

$$0.6\ (\$440{,}000)\ +\ 0.4\ (-\$200{,}000)\ =\ \$184{,}000,$$

which we write above node B in Figure 1.18.

Finally, at node A, the option with the highest EMV is to continue the development of the software. Therefore, we assign the EMV of node A to be the higher of the two EMV values of the branches emanating from the node, i.e., \$184,000, and we cross off the branch corresponding to accepting the offer from Medtech, as shown in Figure 1.18.

We have now solved the decision tree, and can summarize the optimal decision strategy for Bio-Imaging as follows:

Bio-Imaging Optimal Decision Strategy:

- Bio-Imaging should continue the development of the three-dimensional software program and should reject the offer from Medtech.
- If the development effort succeeds, Bio-Imaging should accept the offer from Nugrowth Development.
- If the development effort fails, Bio-Imaging should abandon the project altogether.
- The EMV of this optimal decision strategy is \$184,000.

Sensitivity Analysis

Given the subjective nature of many of the data values used in the Bio-Imaging decision tree, it would be unwise to adopt the optimal decision strategy derived herein without a thorough examination of the effects of key data assumptions and key data values on the optimal decision strategy. Recall that sensitivity analysis is the process of testing and evaluating how the solution to a decision tree behaves in the presence of changes in the data. The data estimates that James had developed in support of the construction of the decision tree were developed carefully, of course. Nevertheless, many of the data values, particularly the values of many of the probabilities, are inherently difficult or even impossible to specify with precision, and so should be subjected to sensitivity analysis. Here we briefly show how this might be done.

One of the probability numbers used in the decision tree model is the probability of winning an SBIR grant with the three-dimensional prototype software. Let us denote this probability by q. James' original estimate of the value of q was $q = 0.70$. Because the "true" value of q is also impossible to know, it would be wise to test how sensitive the optimal decision strategy is to changes in the value of q. If we were to construct a spreadsheet model of the decision tree, we would find that the optimal decision strategy remains unaltered for all values of q less than $q = 0.80$. Above $q = 0.80$, the optimal decision strategy changes by having Bio-Imaging reject the offer from Nugrowth in favor of applying for the SBIR grant with the three-dimensional prototype. If James were quite confident that the true value of q were less than 0.80, he would then be further encouraged to adopt the optimal decision strategy from the decision tree solution. If he were not so confident, he would then be wise to investigate ways to improve his estimate of the value of q.

Another probability number used in the decision tree model is the probability that Bio-Imaging would successfully develop the three-dimensional prototype. Let us denote this probability by p. James' original estimate of the value of p was $p = 0.60$. Because the "true" value of p is also impossible to know, it would be wise to test how sensitive the optimal decision strategy is to changes in the value of p. This can also be done by constructing a spreadsheet version of the decision tree, and then testing a variety of different values of p and observing how the optimal decision strategy changes relative to the value of p. Exercise 1.3 treats this and other sensitivity analysis questions that arise in the Bio-Imaging decision tree.

Decision Analysis Suggests Some Different Alternatives

Note from Figure 1.18 that the EMV of the optimal decision strategy, which is \$184,000, is not that much higher than the EMV of accepting the offer from Medtech, which is \$150,000. Furthermore, the optimal decision strategy, as outlined above, entails substantially more risk. To see this, observe that under the optimal decision strategy, it is possible that Bio-Imaging would realize net profits of \$1,800,000, but they might also realize a loss of $-$\$200,000. If they were instead to accept the offer from Medtech, they would realize a guaranteed net profit of \$150,000. Because the two EMV values of the two different strategies are so close, it might be a good idea to explore negotiating with Medtech to see if they would raise their offer above \$150,000.

In order to prepare for such negotiations, it would be wise to study the product development decision from the perspective of Medtech. With all of the data that has been developed for the decision tree analysis, it is relatively easy to conduct this analysis. Let us presume that Medtech would invest \$1,000,000 in the project (which is the same amount that Nugrowth would have invested). And to be consistent, let us assume that the possible total revenues that Medtech might realize would be the same as those that were estimated for the case of Bio-Imaging accepting financing from Nugrowth, and with the same probabilities (as specified in Table 1.3). If we further assume, to be safe, that Medtech would face the same probability of successful development of the three-dimensional software program, then Medtech's net profits from this project can be represented as in Figure 1.19. In the figure, for example, the net profit of the "high profit" outcome is computed as:

$$\$8{,}850{,}000 = \$10{,}000{,}000 - \$1{,}000{,}000 - \$150{,}000,$$

where the \$10,000,000 would be the total revenue, the \$1,000,000 would be their investment cost, and the \$150,000 would be their offer to Bio-Imaging to receive the rights to develop the product.

The EMV of the node labeled "T" of the tree in Figure 1.19 is computed as:

$$0.20\ (\$8{,}850{,}000) + 0.40\ (\$1{,}850{,}000) + 0.40\ (-\$1{,}150{,}000) = \$2{,}050{,}000,$$

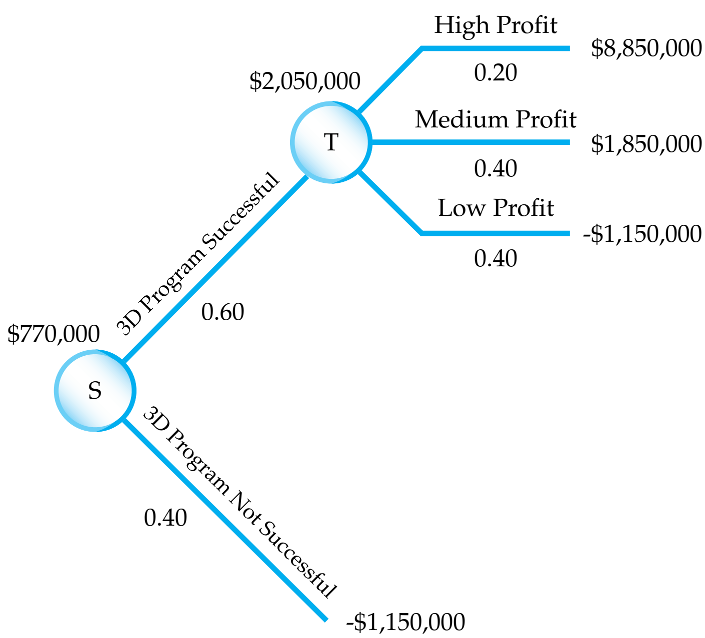
and the EMV of the node labeled "S" of the tree in Figure 1.19 is computed as:

$$0.60\ (\$2{,}050{,}000) + 0.40\ (-\$1{,}150{,}000) = \$770{,}000.$$

Therefore, it appears that Medtech might still realize a very large EMV after paying Bio-Imaging \$150,000 for the rights to develop the software. This suggests that Bio-Imaging might be able to negotiate a much higher offer from Medtech for the rights to develop their three-dimensional imaging software.

One other strategy that Bio-Imaging might look into is to perform the clinical trials to fine-tune the two-dimensional software and apply for an SBIR grant with the two-dimensional software, without attempting to develop the three-dimensional software at all. Bio-Imaging would incur the cost of the clinical trials, which James had estimated to be \$100,000, but would save the \$200,000 in costs of further development of the three-dimensional software. In order to ascertain whether or not this

FIGURE 1.19
Decision tree faced by Medtech.



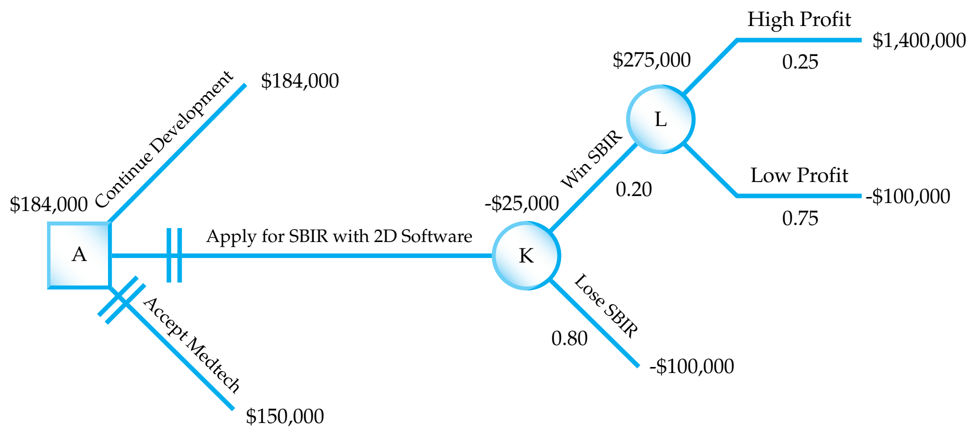


FIGURE 1.20 Evaluation of another development strategy.

might be a worthwhile strategy, we can draw the decision tree corresponding to this strategy, as shown in Figure 1.20.

The decision tree in Figure 1.20 uses all of the data estimates developed by James for his analysis. As shown in Figure 1.20, the EMV of this alternative strategy is −$25,000, and so it is not wise to pursue this strategy.

As the previous two analyses indicate, decision trees and the decision analysis framework can be of great value not only in computing optimal decision strategies, but in suggesting and evaluating alternative strategies.

1.4 THE NEED FOR A SYSTEMATIC THEORY OF PROBABILITY

In the decision tree for Bill Sampras' summer job choice, as well as the decision tree for Bio-Imaging's development strategy, the probability numbers presented in the problem were exactly in the form needed in order to construct and solve the decision tree. This was very convenient. However, in many management contexts, the probability numbers that one needs to solve the problem must be derived and computed using a formal theory of probability, and such a theory will be developed in Chapter 2. To illustrate the need for such a theory of probability, consider the following relatively simple decision problem.

DEVELOPMENT OF A NEW CONSUMER PRODUCT

Caroline Janes is the marketing manager for a consumer products company that is considering whether to produce a new automatic dishwashing detergent called "Suds-Away." To keep this problem simple, let us assume that the market for Suds-Away will either be weak or it will be strong. If the market is strong, the company will make $18 million on Suds-Away, but if the market is weak, the company will lose $8 million. Based on a combination of experience and intuition, Caroline has estimated that there is a 30% chance that the market for Suds-Away will be strong.

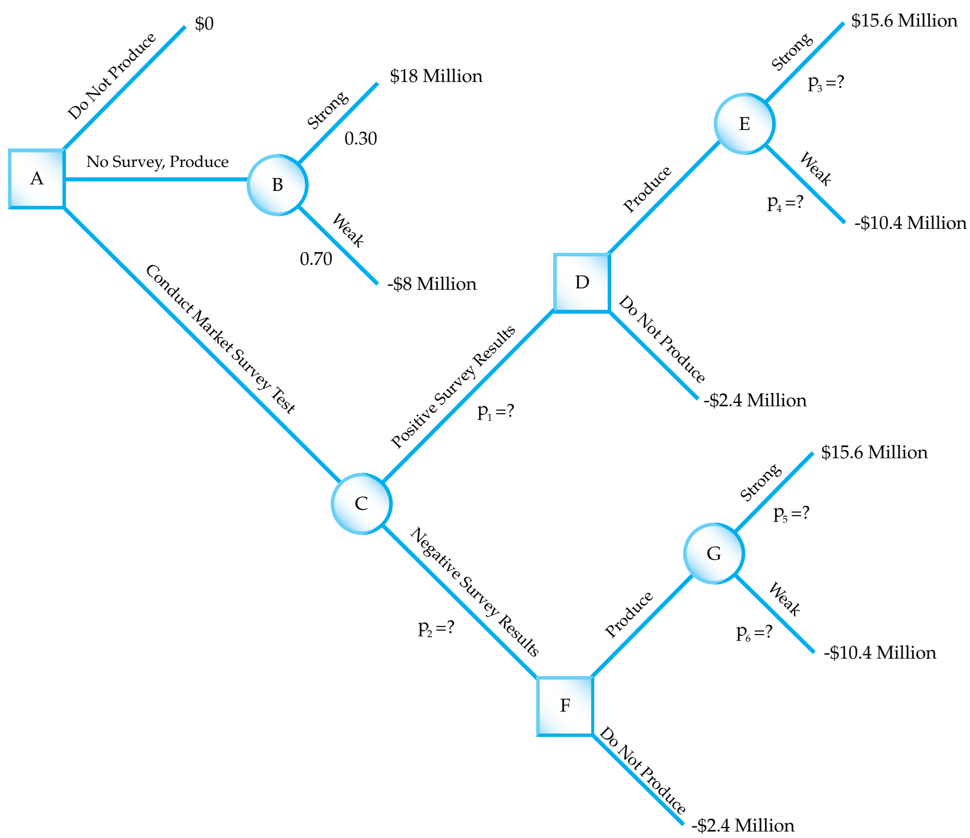
Prior to deciding whether or not to produce Suds-Away, Caroline can conduct a nationwide market survey test of Suds-Away. The cost of the market survey would be $2.4 million. Such market survey tests cannot predict the market for new products with certainty, that is, these survey tests sometimes misread the market for new products. Past results with such surveys indicate that if the market is weak, there is a 10% chance that the test will be positive. Also, if the market is strong, there is a 20% chance that the test will be negative.

Caroline can decide either to not produce Suds-Away, to conduct the market survey test prior to deciding whether or not to produce, or to go ahead with production without conducting such a market survey test.

Figure 1.21 shows the layout for the decision tree for Caroline Janes' decision problem. The immediate decision that Caroline faces is among three different actions: (i) Do not produce Suds-Away; (ii) produce Suds-Away without conducting the market survey test; or (iii) conduct the market survey test prior to deciding whether or not to produce Suds-Away. These three actions are shown as the decision branches emanating from decision node A of the decision tree in Figure 1.21. If Caroline decides to not produce Suds-Away, then her company will realize no revenues, as is shown in the upper branch emanating from node A. If Caroline decides to produce Suds-Away, then the company will either make \$18 million (if the market is strong) or lose \$8 million (if the market is weak). These possibilities and their respective probabilities are shown in the decision tree at event node B.

Suppose that Caroline decides to conduct the nationwide market survey test, which is the bottom branch emanating from node A. The next event that will transpire will be the outcome of the test. The outcome will be either a "positive" or a "negative" market indication for the success of Suds-Away. These two possibilities are shown at event node C of Figure 1.21.

FIGURE 1.21 Caroline Janes' decision tree for the Suds-Away decision problem.



After she receives the information on the outcome of the market survey, Caroline will next have to decide whether or not to produce Suds-Away. Such a decision is shown at decision node D and at decision node E of Figure 1.21. At first glance, it might seem obvious that Caroline should produce Suds-Away if the test results are positive and that she should not produce Suds-Away if the test results are negative. However, the decision to produce Suds-Away at this point depends very much on the accuracy of the market survey test as well as the potential revenue implications of the state of the market for Suds-Away, and so it is prudent to leave this decision in the tree.

Notice that Caroline's decision as to whether or not to produce Suds-Away after receiving the results of the market survey test is drawn in two different places in the decision tree in Figure 1.21. This is because Caroline will face this decision under two different prior sequences of events. She will face this decision if the market survey test outcome is positive; and, of course, she will face the same decision even if the market survey test outcome is negative.

Suppose that the market survey test outcome is positive and that Caroline decides to produce Suds-Away. (This is the upper branch emanating from decision node D in Figure 1.21.) Even though the market survey test is positive in this case, there is still the possibility that the test will misread the market and that the market for Suds-Away might be weak rather than strong. For this reason, we must place an event node, labeled E in Figure 1.21, after Caroline's decision to produce Suds-Away.

Similarly, suppose that the market survey test outcome is negative, and that Caroline decides to produce Suds-Away anyway. This is the upper branch emanating from decision node F in Figure 1.21. Even though the market survey test is negative, there is still the possibility that the test will misread the market and that the market for Suds-Away might be strong rather than weak. Therefore, we must also place an event node, labeled G in Figure 1.21, after Caroline's decision to produce Suds-Away in this case.

The revenue numbers of the final branches in the lower portion of the decision tree in Figure 1.21 are easy to derive. Consider, for example, the case where Caroline has decided to conduct the market survey (at a cost of \$2.4 million), and that the outcome of the test is positive. At decision node D, if Caroline decides to produce Suds-Away, then the company will realize a net contribution to earnings of \$15.6 million if the market is strong. This is because if the market is strong, the company will earn \$18 million directly, but will have expended \$2.4 million on the cost of the market survey test, for a net contribution of \$15.6 million ($15.6 = 18 - 2.4$). This is shown at the end of the upper branch emanating from event node E.

Similarly, if Caroline decides to produce Suds-Away, then the company will realize a net loss in earnings of \$10.4 million if the market is weak. This is because if the market is weak, the company will lose \$8 million directly, and will have also expended \$2.4 million on the cost of the market survey test, for a total loss of \$10.4 million ($10.4 = 8 + 2.4$).

Of course, if Caroline decides not to produce Suds-Away after conducting the market survey test, then the company will lose \$2.4 million, which is the cost of the test. This is shown in the bottom branch emanating from decision node D. Similar logic is used to derive all of the other revenue numbers of the final branches of the decision tree.

In order to solve Caroline's decision problem, we have to insert the relevant probability numbers into the decision tree of Figure 1.21. First consider event node B. We know from the statement of the problem that there is a 30% chance that the market for Suds-Away will be strong, and so the probability of the upper branch emanating from node B is $p = 0.30$. Likewise, the probability of the lower branch emanating from node B is $p = 0.70$.

When we consider the probability numbers for the other branches of the decision tree of Figure 1.21, we run into a bit of difficulty. For example, consider event node C of the decision tree. The two event branches emanating from this node represent the possible events that the market survey test result will be positive (the upper branch) or negative (the lower branch). Let p_1 denote the probability that the market survey test result will be positive. Although we are not given the value of p_1 in the statement of Caroline's decision problem, we need to derive p_1 in order to solve the decision tree. As it turns out, there is a way to derive the value of p_1 based on a more systematic approach to probability theory that is based on the **laws of probability** and the use of **probability tables.** This material will be developed in Section 2.2 and Section 2.3 of Chapter 2.

There are a total of six probability numbers, labeled p_1, p_2, p_3, p_4, p_5, and p_6 in Caroline's decision tree in Figure 1.21, that need to be computed and placed in the decision tree before we can solve the decision tree. All of these probability numbers can be derived and computed using the theory of probability, which will be developed in Chapter 2. In fact, we will revisit Caroline Janes' decision tree in Section 2.3 of Chapter 2, where we will use the laws of probability and probability tables to derive these six probability numbers and then solve Caroline Janes' decision tree.

1.5 FURTHER ISSUES AND CONCLUDING REMARKS ON DECISION ANALYSIS

In this chapter, we have introduced the methodology of decision analysis as a systematic way to analyze a managerial problem under uncertainty. We conclude this chapter with remarks concerning two aspects of the decision analysis methodology: the EMV criterion and considerations of risk, and the issue of non-quantifiable outcomes. Last of all, we discuss some of the many benefits of decision analysis.

The EMV Criterion and the Consideration of Risk

In all of our previous analyses we used the expected monetary value (EMV) criterion to compare various uncertain alternative decisions. The EMV criterion represents an "averaging" approach to compare uncertain alternatives: The EMV is calculated by considering the probabilistic average of all of the possible outcomes. This is a reasonable approach when the range of outcomes is within the realm of normal considerations of risk. However, when the outcomes might represent substantial risk to the decision-maker, the EMV criterion is not so appropriate. This is illustrated below.

Consider a situation where you face a choice between two alternatives as follows. If you choose the first alternative, you will receive \$3 million with certainty. If you choose the second alternative, you will receive \$10 million with probability 0.50, or you will receive \$0 with probability 0.50. The EMV of the first alternative is \$3 million, and the EMV of the second alternative is \$5 million. Although the EMV of the second alternative is greater (by \$2 million) than the EMV of the first alternative, most people (including the authors) would choose the first alternative. This is because the first alternative has less risk: It will assure you of \$3 million. The second alternative is more risky; in fact, there is a 50% chance that you will receive nothing at all if you choose the second alternative. Therefore, for this type of problem, the EMV criterion is not appropriate.

On the other hand, suppose that the above decision problem were faced by a very wealthy individual or a large firm. A wealthy individual or a large firm has the financial resources to take the gamble of the second alternative, and might therefore choose among the alternatives using the EMV criterion.

(As it turns out, there is a method that allows one to formally incorporate risk into decision tree models. The method is called **expected utility theory.** However, the method of expected utility theory is not widely used in practice, and so we do not present it.)

In summary, the EMV criterion is a reasonable criterion for a large number of decision problems, especially those in which the relative outcomes are small compared to the resources of the decision-maker.

Non-quantifiable Consequences

When making any decision, a manager needs to consider consequences that are not easily quantifiable. For example, in the case of Bill Sampras' summer job decision, there are many non-quantifiable consequences of the various choices, such as learning and resumé-building opportunities, the physical attractiveness of the job site, and the "quality" of the job (this may include the level of responsibility, exposure to upper management, etc.).

As it turns out, there are sophisticated analytic tools for incorporating many non-quantifiable aspects of decisions into the decision analysis methodology. However, these methods are also not widely used, and so are not presented.

The Many Benefits of Using Decision Analysis

In addition to determining the optimal decision strategy, decision analysis offers many other benefits. These include the following:

- **Clarity of the decision problem.** By constructing the decision tree, the decision-maker is able to see clearly the structure of the sequence of decisions and uncertain events, and to see the interplay between the decisions and the uncertain events.
- **Insight into the decision process.** By solving the decision tree, the decision-maker is able to see what determines the optimal decision strategy.
- **Importance of key data.** By performing sensitivity analysis on key data values, the decision-maker is able to see which data is more important in determining the optimal decision strategy. This is particularly helpful in suggesting where to invest more time and energy in further data gathering.
- **Other benefits.** Quite often, the entire decision analysis process suggests new ways to think about the decision problem that the decision-maker would not otherwise have thought about.

Many management decision problems are complex and involve a large number of decisions and uncertainties that unfold over time. Decision analysis helps to break a problem into smaller, more manageable pieces. Decision analysis is a tool that can be used to enhance and improve a manager's judgment. In the hands of a good manager, the decision analysis process usually leads to more informed, insightful, and therefore better decision-making overall.

1.6 CASE MODULES

KENDALL CRAB AND LOBSTER, INC.

It was shortly before noon. Jeff Daniels, director of Overnight Delivery Operations at Kendall Crab and Lobster, Inc. (KCL) in Kendall Square, Cambridge, Mass., anxiously watched the Weather Channel on his office television. A fall storm was rapidly moving along the Atlantic coast toward Boston. If the storm front continued to move north at its current speed, the storm would hit Boston at around 5 P.M. However, many such storms change direction and move out to sea before they reach Boston, leaving Boston with only minor precipitation. The weather forecaster predicted a 50% chance that the storm would hit Boston (at around 5 P.M.), and a 50% chance that the storm would move out to sea and miss Boston and the rest of the northern Atlantic states. Jeff Daniels was not the only employee in Kendall Square watching the Weather Channel so attentively. Because there was a chance that Boston's Logan International Airport might have to shut down operations if the storm hit, many business travelers were nervously awaiting further weather information as well. Historically, when storms of this magnitude hit Boston, one in five are accompanied by severely strong winds that force Logan to close down its operations almost immediately.

Kendall Crab and Lobster, Inc.

Kendall Crab and Lobster, Inc. (KCL) was founded in Cambridge, Massachusetts, in 1962, as a crab and lobster wholesale and delivery company for the Boston area. By 1985, KCL had largely eliminated its crab business and had expanded its operations to include on-demand overnight delivery of lobsters to restaurants, caterers, and consumers in the Northeastern United States, with customers from Washington, D.C. to Presque Isle, Maine. By 1998, KCL's annual sales had reached $22 million. KCL attributed its success to great customer service, focused direct-mail marketing and advertising of its product, and the enormous popularity of lobster as a menu item for special occasions. KCL knew that customer service was critical for the success of any business in the food service sector of the economy, and maintaining an excellent reputation for customer service has always been a very high priority.

Jeff Daniels had worked for KCL part-time while a student at the Sloan School of Management at MIT, and had joined KCL's staff full-time after graduation. He rose quickly through the organization to his current position as director of Overnight Delivery Operations, which has been the company's most profitable department. He knew that the senior management were keeping an eye on him, and he had his mind set on becoming a senior vice-president in the next year.

Lobster

Lobster is a very popular menu item. This popularity stems from its exquisitely rich taste as well as its striking appearance, which decorates any dining table beautifully. People typically dine on lobster to celebrate a special occasion, and the experience of eating lobster is fun and exciting. Furthermore, lobster is extremely simple to cook: One simply places the live lobster in a pot of boiling water for 15 minutes, and it is ready to eat!

However, lobster is so perishable that it must be cooked live. After death, an uncooked lobster's meat rapidly deteriorates. For this reason, lobsters must always be transported live, and this adds significantly to their cost. A lobster is prepared for transport by packing the lobster in a corrugated box with an insulating foam insert, and covering the lobster with frozen non-toxic gel packs inside the insulating foam. A lobster can live in this special box for 36 to 48 hours. It is always necessary to transport lobster by overnight air or truck delivery to ensure that it is delivered live to its recipient.

Overnight Delivery Operations

Customers can order lobsters for next-day delivery any time prior to 5 P.M. on the day before delivery. A typical day's orders amount to approximately 3,000 lobsters that need to be delivered to customers. The staff of the KCL Overnight Delivery Operations spend most of their workday processing orders in their ordering and accounting computer systems and packing the lobsters for shipment. At 5:30 P.M., trucks from United Express Overnight Delivery (a prominent overnight package delivery company) pick up the packed lobsters and truck them to their Logan Airport facility. A United Express plane normally takes off from Logan Airport with the lobsters (and other packages from other Boston clientele) at 6:30 P.M. and flies the packages to United Express' central processing and sorting facility near Washington, D.C. At this facility, all packages (including the lobsters) are sorted and classified by their final destinations, and transported by air and then local trucks during the night, typically arriving at their destination by 10:30 A.M. the next day. The price that KCL charges its customers for delivered lobsters depends on the lobster's size, but typically this price is $30 per lobster, which includes all transportation costs. When KCL ships a lobster via United Express, its unit contribution to earnings is close to $10 per lobster.

If, for any reason due to weather, KCL is not able to deliver a customer's lobster order, it is KCL's policy to notify the customer by telephone (or FAX), to refund the purchase price of the lobster, and to give the customer a $20 discount coupon per lobster, which can be used towards the purchase of lobster from KCL at any time in the next twelve months. When this happens, customers are typically disappointed that they will not receive their lobster order, but they usually understand the relationship between delivery and weather conditions, and they appreciate the discount coupon. Marketing data has shown that approximately 70% of customers who receive such coupons eventually redeem them.

Changes in Operations Due to Weather Activity

Serious coastal storms that have the potential to close down Logan International Airport hit Boston about ten times per year. However, these storms virtually never threaten inland transportation operations on the ground or in the air. For this reason, KCL has often in the past relied on the services of Massachusetts Air Freight (MAF), which operates out of Worcester, Massachusetts (approximately 50 miles west and inland from Boston) for assistance in the transport of lobsters when adverse weather closes down Logan International Airport. If contacted before 5:30 P.M., MAF will pick up the packaged lobsters from KCL in Kendall Square, deliver them by truck to their airport in Worcester, Mass., and then fly them to United Express Overnight Delivery's sorting facility in Washington, D.C., whereupon United Express will take over the further routing and eventual delivery of the lobsters to customers. The costs associated with using MAF to transport lobsters to United Express' sorting facility in Washington, D.C. are typically quite high. According to Jeff's spreadsheet records, the additional transportation cost of using MAF was $13 per lobster in roughly 67% of the times that MAF was used, and was $19 per lobster in the remaining 33% of the times that MAF was used.

The other option that KCL would have if the storm were to force Logan Airport to close would be to simply cancel all orders for delivery to customers for the next day. This option involves notifying customers by telephone and informing them that they will be receiving a coupon in the mail for each lobster ordered. Also, if the lobsters have already been packed for transport, they would have to be unpacked and returned to their holding tanks for processing the following day. If the lobsters were not already packed, the incremental cost of canceling the orders is approximately $1.00 per lobster. If the lobsters were already packed, the cost is instead $1.25 per lobster, since most of the packaging materials are reusable.

With the storm approaching, Jeff faced one other immediate option: He could use a truck delivery company to deliver the lobsters. If Jeff were to make arrangements with Eastern Parcel Delivery (EPD), a regional truck package delivery company, by noon, they would hold trucks for KCL, and pick up the lobsters at 5:30 P.M. for delivery to all customers by late the next morning. EPD was very reliable as a trucking package delivery service, but there were two problems with using EPD. First, KCL would need to commit to using trucks by noon, well before they would know additional information about the storm. Second, truck delivery via EPD was more costly than delivery via air from United Express. EPD's delivery fees typically depended on the total distance that their trucks have to travel. However, because KCL would still be accepting orders from new customers until 5 P.M., the costs of using EPD could not be known at noon with any certainty. Depending on the distance the trucks had to travel, the cost of truck delivery could range from $2 to $4 higher than normal air delivery per lobster. A quick scan of the data in his spreadsheet on previous instances when KCL arranged transportation from EPD showed that in 50% of the times EPD was used, the average additional transportation cost was around $4 per lobster (above the normal cost of delivery by air). In another 25% of the cases, the average additional transportation cost was around $3 per lobster. In the remaining 25% of the cases, the average additional transportation cost was only $2 per lobster.

Many Options to Choose From

If Jeff were to choose to transport the lobsters by truck using EPD, he would have to make the arrangements almost immediately. If he chose not to use EPD, he could wait and see if the storm hit Boston and if the storm forced Logan to close down. If the storm were to hit Boston, it would do so at around 5 P.M. and Logan Airport would either close immediately or not at all, depending on the severity of accompanying winds. If the storm were not to hit Boston and/or the storm were not strong enough to force Logan to close, Jeff would presumably know this by 5:30 P.M., and he could then go ahead with KCL's regular plan of having United Express Overnight Delivery pick up the lobsters for air transport out of Logan Airport. If the storm were to hit Boston and close down Logan Airport, Jeff would also know this by 5:30 P.M., and he would then face the decision of whether to cancel the customers' deliveries altogether, or to transport the lobsters via MAF to Worcester and on to United Express' sorting facility in Washington, D.C., where the lobsters would then be incorporated into United Express' delivery operations.

Regardless of which options Jeff chose, he knew that on Friday he would have to explain his decision to senior management at their weekly operations review meeting. As he sat down to do a decision tree analysis, he also thought it would be an excellent idea to standardize the decision tree methodology so that it could be used semi-automatically in the future. Standardizing the methodology made good sense as a means of keeping KCL's operations consistently efficient. As a side benefit, Jeff hoped that this would also impress the senior managers at the meeting.

Assignment:

(a) Structure the delivery operations decision problem as a decision tree.

(b) Solve for Jeff Daniels' optimal decision strategy.

BUYING A HOUSE

Debbie and George Calvert are thinking of making an offer to purchase a house in Shaker Heights, Ohio. Both George and Debbie saw the house this morning and fell in love with it. The asking price for the house is $400,000, and it has been on the market for only one day. Their broker told them that there were more than 20 potential buyers who saw the house that day. She also added that another broker told her that an offer on the house was going to be presented by that broker this afternoon. Their broker has advised them that if they decide to make an offer on the house, they should offer very close to the asking price of $400,000. She also added that if there are competing offers on the house that are close in value, then it is common practice for the seller to ask the potential buyers to submit their final offers the following day.

Trying to be objective about this decision, Debbie has decided to construct a decision tree to help her with this decision. She has assumed that the "fair market value" of the house under consideration is $400,000. She has assigned an "emotional value" of $10,000 if she and George are successful in purchasing the house. That is, whereas the fair market value of the house is $400,000, the house is worth $410,000 to Debbie and George. Thus, if they were to be successful in purchasing the house for $390,000, the value of this outcome would be $20,000. Of course, if they were not successful in purchasing the house, the value of this outcome would be simply $0. Debbie has also assigned a probability of 0.30 that they will be the only bidders on the house.

Debbie has decided to consider making one of only three offers: $390,000, $400,000, or $405,000. She estimates that if they are the only bidders, the probability that an offer of $390,000 is accepted is 0.40, the probability that an offer of $400,000 is accepted is 0.60, and the probability that an offer of $405,000 is accepted is 0.90.

If, however, there are other bidders, Debbie assumes that the seller will ask them to submit a final offer the following day. In such a scenario, she will then have to rethink what to do: She can withdraw her offer, submit the same offer, or increase her offer by $5,000. She feels that in the event of multiple bids, the probability that an offer of $390,000 is accepted is 0.20, the probability that an offer of $395,000 is accepted is 0.30, the probability that an offer of $400,000 is accepted is 0.50, the probability that an offer of $405,000 is accepted is 0.70, and the probability that an offer of $410,000 is accepted is 0.80.

Assignment:

(a) Structure Debbie and George's problem as a decision tree.

(b) Solve for Debbie and George's optimal decision strategy.

THE ACQUISITION OF DSOFT

Polar, Inc. and ILEN are the two largest companies that produce and sell database software. They each have been negotiating to buy DSOFT, the third largest company in the database software market. Polar currently has 50% of the world market for database software, ILEN has 35%, DSOFT has 10%, and there are several other smaller companies that share the remaining 5% of the market for database software. Financial and market analysts at Polar have estimated the value of DSOFT to be $300 million in net worth.

Throughout the preliminary negotiations, DSOFT has made it clear that they will not accept a purchase offer below $300 million. Jacob Pratt, the CEO of Polar, figures that acquiring DSOFT will make Polar the dominant player in the industry as Polar would then have 60% of the market. In addition, Jacob knows that DSOFT has been developing a new product that has tremendous earnings potential. Jacob has estimated that the new product would increase the net worth of DSOFT by an additional $300 million with probability 0.50, by an additional $150 million with probability 0.30, or have no impact on net worth with probability 0.20.

To simplify matters, Jacob has decided to consider three possible strategies regarding the possible purchase of DSOFT: (i) Make a "high" offer of $400 million; (ii) make a "low" offer of $320 million; or (iii) make no offer at all. If he pursues this third strategy (making no offer), Jacob is certain that ILEN will buy DSOFT. If Polar makes an offer to DSOFT (either a "high" or a "low" offer), Jacob figures that ILEN will increase the offer further. He is uncertain about what ILEN will offer in this case, but he has made the following intelligent estimates of possible outcomes: ILEN would increase Polar's offer by 10% with probability 0.30, by 20% with probability 0.40, or by 30% with probability 0.30. If ILEN were to make such an offer, Jacob would then need to decide whether he would make a final offer to DSOFT. His thinking is that after ILEN makes a counter-offer, Polar would either withdraw from the bidding, match the counter-offer, or make a final offer at 10% above ILEN's counter-offer. If Polar's offer and ILEN's offer are identical, Jacob estimates that the probability that DSOFT will accept Polar's final offer is 0.40 ; however, if Polar counters with an offer which is 10% higher than ILEN's offer, Jacob estimates that DSOFT will accept Polar's final offer with probability 0.60.

Assignment:

(a) Structure Polar's acquisition offer problem as a decision tree.

(b) Solve for the optimal decision strategy for Polar.

NATIONAL REALTY INVESTMENT CORPORATION

National Realty Investment Corporation is a firm that is principally engaged in the purchase, development, management, and sale of real estate properties, particularly residential properties. It is currently evaluating whether or not to purchase the old Bronx Community Hospital, located in the East Bronx of New York City in a neighborhood undergoing rapid growth and rejuvenation. The Bronx Community Hospital operated unprofitably until 1991 when it closed and filed for bankruptcy. National Realty is considering purchasing the building, converting it to apartment units, and operating it as an apartment complex.

The hospital and its property are currently being offered for sale. National has performed an extensive evaluation of the site and neighborhood, and has projected the value of the property to be $2 million. After considering its size and location, National thinks that this is an attractive price and is considering offering to purchase the property at this price. Carlos Melendez is in charge of the hospital project for National. Carlos has thought that this property could be converted into 96 "low-income" apartments. The location seems ideal for such a conversion, and with the help of state agencies that would cooperate with (and subsidize) this type of development, the remodeling could be quite a profitable investment.

The New York Housing and Development Agency (NYHDA) has been administering the Low-Income Housing plan (LIH) for many years, in an effort to accommodate families in need of affordable housing. Because of the high demand for housing in New York City, rents have been increasing at rates higher than 10% per year in many neighborhoods, leaving large numbers of families without affordable housing. For this reason, the NYHDA started the LIH plan, which provides financial incentives for real-estate developers to become involved in the rehabilitation of properties for low-income families. Under normal circumstances such projects might not be appealing to developers, since other investments might yield a higher return (with similar risk). With LIH financial incentives, which represent large subsidies to the developer from the NYHDA, these projects become attractive enough to warrant investment. The NYHDA takes many things into consideration before approving any project under the LIH plan, and all applications are carefully reviewed by different area specialists. The LIH plan subsidies represent long term investments of large amounts of money. Therefore, the NYHDA takes great pains to ensure that all projects are fully completed and that they serve their original purpose of providing housing for low-income families.

Carlos Melendez needs to decide whether National should offer to purchase the hospital or not; however, the decision process is more complex than this, since there are many factors that are uncertain. One of the key issues is whether the 96-unit renovation plan would be granted approval by the NYHDA. In some instances when a developer applies for NYHDA subsidies under the LIH plan, the NYHDA evaluation process can take as long as five months. Unfortunately, National Realty has to make some decisions quite soon.

According to Carlos, some of the uncertainty concerning the approval of a potential application for the hospital property could be resolved if he could postpone his decision for at least one month, until after the coming November elections, when New York City voters will elect a new mayor. One of the candidates is fully supportive of the LIH plan, and she has been encouraging more low-income housing for quite some time. The other candidate has not been supportive of the low-income housing concept or the LIH plan of subsidizing private development. Obviously, the chances of approval of an application for the hospital property to qualify for LIH plan subsidies would increase or decrease according to the outcome of the elections. Carlos thought that if he were to wait until after November to make a decision on the property, he would have a better idea of whether his LIH plan application was going to be approved. Unfortunately, there is always the risk that the property would be purchased by some other buyer if National waited until after the elections. Carlos therefore needs to decide whether to make an offer now or to wait until after the elections and risk that the property would already be sold.

If and when National were to offer to purchase the hospital, they must also offer a non-refundable deposit of 10% of the purchase offer (in this case, $200,000). Once National has made an offer to purchase the hospital, National can then apply for the LIH plan subsidies. After 60 days from the date that the offer and deposit are drawn up, National must either complete the offer to buy the property, in which case the remaining 90% of the value of the property is paid, or withdraw the offer, forfeiting their 10% deposit.

There is no guarantee, unfortunately, that National's LIH subsidy application would be processed and decided upon within the 60-day period National has before making a final decision. (In the past, the NYHDA approval process has taken anywhere from one month to five months.) If the NYHDA were to decide on National's application within the 60-day time period, then this would make National's evaluation process easier. However, there is a distinct possibility that if National were to offer to purchase the hospital (to develop it for low-income housing), they would not hear from NYHDA regarding the status of their application before the 60-day limit that the hospital trustees have. National would have to decide either to withdraw their offer (and lose their 10% deposit) or to purchase the property without knowing whether their application for LIH plan subsidies is going to be accepted.

Initially, Carlos Melendez thought that the hospital renovation project would be profitable only if it received the LIH plan subsidies from the NYHDA. Renovation as well as maintenance costs on the property were projected to be quite high, and consequently higher rents would ordinarily have to be charged to tenants to cover these costs. However, because of the location of the property in the East Bronx, Carlos did not think this would be possible. At the same time, because of all of the new development and construction that has been going on in the neighborhood, Carlos thought that medium-income families might find the renovated apartments attractive and be willing to pay the higher rents. This might make the renovation project cost-effective even if it were not approved for LIH development (and subsidies).

Analyzing the Problem

Carlos Melendez sat at his desk trying to conceptualize his problem. What were the key decisions? What were the key uncertainties? What was the timing of decisions? What was the timing of the resolution of uncertainty? What data would he need in order to make an intelligent, defensible decision? After some hard thinking, he drew up the following list of uncertainties for the hospital renovation project.

- The chance that the 96-unit LIH plan would be approved.
- The chance that the LIH application decision would be delayed beyond 60 days after submission.
- The chance that the mayoral candidate who is favorable to low-income housing would be elected.
- The chance that the hospital property would still be available for purchase if National waited until after the November election.
- The chance that the LIH plan would be approved, given that the outcome of the mayoral election was favorable.

Valuing Uncertainty

Carlos reviewed some past history of LIH applications and estimated the probability that the application decision would be delayed beyond 60 days to be 0.30. Carlos then mused that the probability that the outcome of the mayoral election is favorable is 0.60. Given that the outcome of the mayoral elections were favorable, Carlos estimated that the probability of the LIH plan to be approved is 0.70; if the outcome were unfavorable, the probability of approval is 0.20. Finally, Carlos estimated the probability that the property would still be available for purchase after the election (if National took no action) to be 0.80.

The Value of the Two Development Alternatives

Carlos next went to work on the financial analysis of the development project. Armed with financial numbers, cost estimates, and a business calculator, Carlos estimated that the value of the development project under the LIH plan would be $428,817. This value includes the purchase cost of the property, the cost of renovations, annual maintenance costs, annual rental incomes, and applicable tax shelters. (The details of the computation of this number are presented in the appendix for students who are curious to see how these sorts of computations are developed. However, the details of the computations are not needed in order to do the assignment below.)

If LIH plan approval were not granted for the project, National would lose some of the attractive subsidies and tax shelters offered by NYHDA. On the other hand, there would be no restriction on the percentage of low-income tenants that could occupy the building (nevertheless, not too many high- or medium-income families would be attracted to the neighborhood). Carlos estimated that the value of the development project in the absence of LIH approval would be $42,360. Once again, the details of the computation of this number are presented in the appendix for the interested reader.

Assignment:

(a) Construct a spreadsheet model of the decision problem, and solve for the optimal decision strategy.

(b) Carlos assumed that the probability of LIH approval given a favorable election outcome was $p = 0.7$. Perform sensitivity analysis on this probability, letting p vary in the range between 0.6 and 0.8. In what way, if at all, does the optimal decision strategy change?

(c) (Optional Challenge!) According to the appendix, Carlos assumed a discount rate of $\beta = 7\%$. Perform sensitivity analysis on the discount rate, letting β vary in the range between 6% and 9%. In what way, if at all, does the optimal decision strategy change?

(d) What is your overall recommendation?

Appendix: Computation of the Value of the Development Project

In this appendix we illustrate how the value of the development project is computed. In order to evaluate the project, Carlos must properly account for income from the project, which comes from rents on the units, and costs, including renovation costs, maintenance costs, and taxes.

Estimating Rental Income for the Hospital Renovation Project

If a real estate development project is approved under the LIH plan, a specified percentage of the building must be occupied by low-income families. Tenants have been classified into three main groups according to their level of income. The first are class A tenants, which have an annual income of over $20,000 but less than $30,000. The second class are called class B tenants, who have an annual income between $15,000 and $20,000. Class C tenants are those with an income of $15,000 or less.

According to the specific housing conditions, the NYHDA requires that 20% of all tenants in a building supported by the LIH plan should belong to class A, 30% to class B, and 50% to class C. Once the LIH plan approval is granted, the only requirement that has to be satisfied is to maintain this distribution of tenants in the buildings. This distribution is important in determining what the investment return of the project will be, since this implies different levels of rents according to the class of tenant. On average, the actual amount of monthly rent charged to the tenants plus the subsidy paid by the agency (which accounts for an additional 20% of what the tenant pays) is: $685 for a class A tenant, $484 for a class B tenant, and $418 for a class C tenant.

If the LIH plan approval is not granted for the project, there is obviously no restriction on the percentage of low-income tenants that could occupy the building. However, there would also not be too many medium- or high-income families that would be attracted to the neighborhood. Carlos has estimated that the average monthly rent in this case would be $520 per rental unit.

Costs of Renovation

Carlos has been working with some of the building planners at National and has developed a good feel for renovation costs. He has estimated that the cost of the renovation would be about \$1,152,000, which works out to \$12,000 per apartment. However, this cost would be lower if the project was approved for LIH subsidies. If their project was approved, National would receive architectural and other technical expertise for free, as well as subsidies for material and labor costs, except for windows and window and door frames, which have to be purchased at market value. Carlos has figured that the total cost would be about 20% lower if the LIH plan was approved.

Annual Maintenance Costs

Maintenance costs for apartment buildings can be divided into two categories, general costs and unit costs. General maintenance costs include costs associated with operating the entire building, such as the supervisors' salaries, the cost of operating the elevators, etc. Carlos estimated this cost to be \$56,000 per year for the project.

Unit maintenance costs are the costs of maintaining the individual apartments, such as utility costs (gas, water, electricity) as well as routine repairs in the apartments. Without LIH approval, this cost would be close to \$75,000 per year. With LIH approval, subsidies of utility costs would reduce this cost to \$60,000 per year.

Taxes

Most of the real estate investments of this dimension are taxed at the annual rate of 45%. However, if granted LIH approval, National would be entitled to a tax rebate of 5%, reducing their effective tax rate to 40%.

Time Horizon

In order to estimate the cash flows of the hospital project, Carlos used a 30-year planning horizon.

Valuing Cash Flows over Time

Suppose that there is an investment opportunity that will pay you \$10,000 next year and \$17,000 the year after, once you make an initial investment this year of \$20,000. Then the total cash flow from the investment would be

$$\$7{,}000 = -20{,}000 + 10{,}000 + 17{,}000.$$

However, this cash flow analysis assumes that \$1 received today is no different than \$1 received next year. This is obviously not true. Almost all people prefer \$1 today to \$1 next year, if for no other reason than the fact that they could put the \$1 in a savings account and earn interest on it. Therefore cash flows in the future are valued less than cash flows in the present. Financial calculations take this into account by discounting cash flows in the future, using a *discount rate.* For example, at a discount rate of $\beta = 0.07 = 7\%$ per year, the value of the above investment would be

$$\$4{,}194 = -20{,}000 + \frac{1}{(1.07)}10{,}000 + \frac{1}{(1.07)^2}17{,}000.$$

Notice that instead of weighting all future cash flows the same, we have discounted each year's cash flow by the factor $0.9346 = 1/1.07$. The number 1.07 is computed as

$1 + \beta$ where β is the discount rate, and so $1 + \beta = 1 + 0.07 = 1.07$. Cash flows that occur t years into the future are discounted using the factor $\frac{1}{(1+\beta)^t}$. More generally,

> The value of a cash flow that pays K_0 now, K_1 in one year, K_2 in two years, . . . , K_t in year t is:
>
> $$\text{Value} = K_0 + \frac{K_1}{1+\beta} + \frac{K_2}{(1+\beta)^2} + \cdots + \frac{K_t}{(1+\beta)^t}$$
>
> where β is the discount rate used in the cash flow analysis.

This way of computing cash flows over time is called **NPV** or **Net Present Value** analysis, and also goes by the name **discounted cash flow** analysis.

In computing the cash flows over time, Carlos used a discount rate of 7%, which is a fairly standard rate to use for this sort of computation. He computed the value of the cash flows of the hospital project under the LIH plan as follows:

The average monthly rent from each of the 96 units is

$$\$491.20 = 0.20 \cdot 685 + 0.30 \cdot 484 + 0.50 \cdot 418,$$

which accounts for the breakdown of rents by income class.

The total income each year (after taxes) is

$$\$269{,}917 = (1.0 - 0.40)(12 \cdot 96 \cdot 491.2 - 56{,}000 - 60{,}000),$$

which accounts for rental income as well as annual maintenance costs, and a tax rate of 40%.

Using a the discount rate of $\beta = 0.07$ on a cash flow of \$269,917 every year for the next 30 years, Carlos computed:

$$\$3{,}349{,}417 = \sum_{t=1}^{30} \left(\frac{1}{1.07}\right)^t \cdot 269{,}917.$$

The overall value of the project after taxes is therefore:

$$\$427{,}817 = -2{,}000{,}000 - (1.0 - 0.20) \cdot 1{,}152{,}000 + 3{,}349{,}417.$$

This last computation accounts for the purchase cost of the property (\$2,000,000) and the cost of renovation (\$1,152,000), less 20% due to LIH subsidies.

Without LIH approval, using a similar analysis, the value of the project would be

$$\$42{,}360 = -2{,}000{,}000 - 1{,}152{,}000$$
$$+ \sum_{t=1}^{30} (1 - 0.45) \cdot \left(\frac{1}{1.07}\right)^t \cdot (12 \cdot 96 \cdot 520 - 56{,}000 - 75{,}000).$$

1.7 EXERCISES

EXERCISE 1.1 Mary is organizing a special outdoors show which will take place on August 15. The earnings from the show will depend heavily on the weather. If it rains on August 15, the show will lose \$20,000; if it is sunny on August 15, the show will earn \$15,000. Historically, the likelihood of it raining on any given day in mid-August is 27%. Suppose that today is July 31. Mary has the option of canceling the show by the end of the day on July 31, but if she does so, she will then lose her \$1,000 deposit on the facilities.

(a) What is Mary's optimal decision strategy?

(b) Suppose that Mary can also cancel the show on August 14, but if she waits until then to do so, she must pay a fee of \$10,000. The advantage of waiting until August 14 is that she can listen to the weather forecast for the next day on the local news station. According to station records, the weather was forecast to be sunny 90% of the days in mid-August in previous years. Also, when the weather was forecast to be sunny, it turned out to be sunny 80% of the time. When the weather was forecast to be rainy, it turned out to be rainy 90% of the time. What is Mary's optimal decision strategy in this case?

EXERCISE 1.2 The Newtowne Art Gallery has a valuable painting that it wishes to sell at auction. There will be three bidders for the painting. The first bidder will bid on Monday, the second will bid on Tuesday, and the third will bid on Wednesday. Each bid must be accepted or rejected that same day. If all three bids are rejected, the painting will be sold for a standing offer of \$900,000. Newtowne's chief auctioneer's estimates for the bid probabilities are contained in Table 1.5. For example, the auctioneer has estimated that the likelihood the second bidder will bid \$2,000,000 is $p = 0.90$.

(a) Formulate the problem of deciding which bid to accept as a decision tree.

(b) Solve for the optimal decision strategy.

EXERCISE 1.3 Set up a spreadsheet to analyze the following issues arising in the Bio-Imaging development strategies example:

(a) How does the optimal decision strategy change relative to the probability of successful development of the three-dimensional computer program? Recall that in James' decision tree model, he estimated the probability of the successful development of the three-dimensional computer program to be $p = 0.60$. To answer this question, let p denote the probability of successful development of the three-dimensional computer program. Then let p vary between $p = 0.0$ and $p = 1.0$ in your spreadsheet, and observe how the optimal decision strategy changes. What do you notice?

(b) Suppose that James' estimates of the total revenues to Bio-Imaging under the "high profit" scenarios are too low. Suppose that Bio-Imaging's total revenues would be \$15,000,000 instead of \$3,000,000 under the "high profit" scenario if three-dimensional software is successful and they win the SBIR grant, and that their total revenue under the "high profit" scenario would be \$50,000,000 rather than \$10,000,000 if they were to accept the financing offer from Nugrowth. How would their optimal decision strategy change?

TABLE 1.5
Estimates of the probabilities of bids by the three bidders.

Amount of Bid	*Bidder 1 (Monday)*	*Bidder 2 (Tuesday)*	*Bidder 3 (Wednesday)*
\$1,000,000	0.0	0.0	0.7
\$2,000,000	0.5	0.9	0.0
\$3,000,000	0.5	0.0	0.0
\$4,000,000	0.0	0.1	0.3

(c) How does the optimal decision strategy change relative to a change in the probability of the "high-profit" scenario? What additional assumptions do you need to make in order to answer this question?

EXERCISE 1.4 Anders and Michael were classmates in college. In their spare time while undergraduates, they developed a software product that regulates traffic on internet sites. Their product uses very imaginative and original ideas, and they have applied for a patent. They estimate that there is an 80% chance their patent will be approved by the US Patent Office.

Anders and Michael have also formed a start-up company called ITNET, and they have started to market their software product. Last month, they presented some of their ideas tof Singular, Inc., the dominant player in this growing market, after Singular had signed a confidentiality agreement with ITNET that ITNET's lawyer had prepared.

Yesterday, Singular announced a new software product that seemed suspiciously similar to the one that Anders and Michael have developed. Anders' first reaction was to plan to sue Singular immediately. However, Michael felt that they should wait until they have received notification of their patent, which is still pending before the U.S. Patent Office. Michael reasoned that their case would be much stronger if they had a patent for their product.

Suppose that Anders and Michael have a 90% chance of winning a lawsuit against Singular if their patent application is approved, and that they still have a 60% chance of winning such a lawsuit even while their patent application is pending (because Singular had signed the confidentiality agreement). However, if their patent application is not approved, suppose that the chance of winning the lawsuit would drop to 40%.

Anders feels that if they sue Singular immediately, there is a 70% chance that Singular would settle out of court for $400,000, and a 30% chance that Singular would not settle out of court. If they win the lawsuit, their settlement would be $1 million. However, they estimate that the legal costs of going to court would be $100,000.

(a) Structure ITNET's problem of whether or not to sue Singular as a decision tree.

(b) Solve for the optimal decision strategy.

EXERCISE 1.5 Javier Peña has always been interested in financing artistic projects. He has recently been offered two financing opportunities in the fashion industry: financing a new line of avant-garde youth fashions designed by Jorge Vera, and financing a line of business attire designed by Paolo Ricci. Javier has had a lot of past experience with these two designers, and has observed that 20% of Vera's fashion lines are "hits" and 80% of them are "misses." Furthermore, Ricci's fashion lines are "hits" 30% of the time, and are "misses" 70% of the time.

Javier's net liquid assets amount to $750,000. As a result, he can afford to finance at most one of the two fashion lines. However, he does have the option of pre-testing at most one of the fashion lines at the upcoming design show in San Francisco, before deciding which, if any, fashion line he would like to finance for the entire U.S. market for the fall fashion season. The costs and revenue associated with the two fashion lines are given in Table 1.6.

Javier has observed, based on previous years, that of the avant-garde fashion lines that were hits nationwide, 80% were hits in the San Francisco pre-test; of the avant-garde fashion lines that were misses nationwide, 40% were hits in the San

TABLE 1.6 Costs and revenue for the two fashion lines. Net costs are given for pre-testing the lines in San Francisco.

Fashion Line:	*Jorge Vera (avant-garde)*	*Paolo Ricci (business attire)*
Net cost of San Francisco pre-test	\$200,000	\$75,000
Additional cost of U.S. production of line after a San Francisco pre-test	\$500,000	\$275,000
Cost of U.S. production if not pre-tested in San Francisco	\$600,000	\$325,000
Revenue if fashion line is a "hit"	\$4,000,000	\$1,000,000
Revenue if fashion line is a "miss"	\$300,000	\$100,000

TABLE 1.7 James McGill's remaining lifetime in years.

Tumor	*Remove Tumor*	*Leave Tumor*
Benign	5	8
Malignant	5	1

Francisco pre-test. Of the business attire fashion lines that were hits nationwide, 90% were hits in the San Francisco pre-test; of the business attire fashion lines that were misses nationwide, 60% were hits in the San Francisco pre-test. While Javier may find pre-test results useful, he knows the accuracy of this kind of test is not high enough to compel him in all cases to act in accordance with the pre-test results. In any event, Javier is willing to act on the basis of expected monetary values.

(a) Develop a decision tree to assist Javier in deciding what to do.

(b) What probabilities need to be computed in order to solve the decision tree?

(c) After reading Chapter 2, compute the necessary probabilities, and solve for Javier's optimal decision strategy.

EXERCISE 1.6 James McGill, age 68, was recently diagnosed with a particular type of brain tumor, and was referred to Dr. Mitchell Zylber, chief of surgery at the university hospital, for further evaluation. This type of tumor is benign in 50% of cases and is malignant in the other 50% of cases. James McGill's remaining lifetime will depend on the type of tumor (benign or malignant) and on the decision whether or not to remove the tumor. Table 1.7 shows estimates of James McGill's remaining lifetime according to the most up-to-date information known about this type of tumor.

Dr. Zylber could perform exploratory surgery prior to the decision whether or not to remove the tumor, in order to better assess the status of the tumor. Exploratory surgery is known to indicate a benign tumor 75% of the time, if the tumor is indeed benign. The surgery is known to indicate a malignant tumor 65% of the time, if the tumor is indeed malignant. Exploratory surgery itself is dangerous: there is a 5% chance that patients with profiles like James McGill's will not survive such surgery due to complications from anesthesia, etc.

If no exploratory surgery is performed, James McGill must decide whether or not to have the tumor removed. And if exploratory surgery is performed, James must decide whether or not to have the tumor removed based on the results of the exploratory surgery.

(a) Draw the decision tree for this problem.

(b) What probabilities need to be computed in order to solve the decision tree?

(c) After reading Chapter 2, compute the necessary probabilities, and solve for the decision strategy that maximizes James McGill's expected lifetime.

(d) James McGill's children are expected to have children of their own within the next two or three years. Suppose James McGill wants to maximize the probability that he will live to see his grandchildren. How should this affect his decision strategy?

(e) What ethical questions does this medical problem pose?