Aufgaben:

① $\left[4\sqrt[3]{x^5}-4\cdot e^x+\sin(x)\right]' =$

$= 4\cdot\frac{5}{3}x^{2/3}-4\cdot e^x+\cos(x) =$

$= \frac{20}{3}\sqrt[3]{x^2}-4e^x+\cos(x).$

$\sqrt[3]{x^5}=(x^5)^{1/3}=x^{5/3}$

$\sqrt[3]{\Delta}=\Delta^{1/3}$

$(x^{5/3})'=\frac{5}{3}\cdot x^{\frac{5}{3}-1}$

$=\frac{5}{3}x^{2/3}$

$\sqrt[n]{x^m}=x^{\frac{m}{n}}$

$\left[2\cdot \overbrace{x^2}^{f}\cdot \overbrace{\ln(x)}^{g}\right]'=$

$2\cdot\left[2x\cdot\ln(x)+x^{\cancel{2}}\cdot\frac{1}{\cancel{x}}\right]=$

$4x\ln(x)+2x.$

$(f\cdot g)'=f'\cdot g+f\cdot g'$

$\left[\frac{10x}{x^2+1}\right]'=\frac{10\cdot1(x^2+1)-10x\cdot(2x+0)}{(x^2+1)^2}$

$=\frac{10x^2+10-20x^2}{(x^2+1)^2}$

$=\frac{10-10x^2}{(x^2+1)^2}=\frac{10(1-x^2)}{(x^2+1)^2}$

$\left(\frac{f}{g}\right)'=\frac{f'\cdot g-f\cdot g'}{g^2}$

$\left[3\cdot e^{-4x}\right]'=3\cdot e^{-4x}\cdot(-4)=-12\cdot e^{-4x}$

$e^{(-4x)}$

$\left[f(\underbrace{g(x)}_{-4x})\right]'=f'(g(x))\cdot g'(x)$

$f=e^{\cdots},\ f'=e^{\cdots}$

$(-4x)'=-4$

$\left[\sin^2(2x-4)\right]'=$

$\sin^2(\alpha)=\sin(\alpha)\cdot\sin(\alpha)$

$=(\sin(\alpha))^2$

a) Produktregel: $=\left[\sin(2x-4)\cdot\sin(2x-4)\right]'$

$=\ldots$

b) Kettenregel: $\left[\left(\sin(2x-4)\right)^2\right]'=2\cdot\sin(2x-4)\cdot\left(\sin(2x-4)\right)'$

äußere Fkt

innere Fkt

$=2\cdot\sin(2x-4)\cdot\cos(2x-4)\cdot2$

$$[\sin(\omega t)]' = \cos(\omega t)\cdot\omega$$

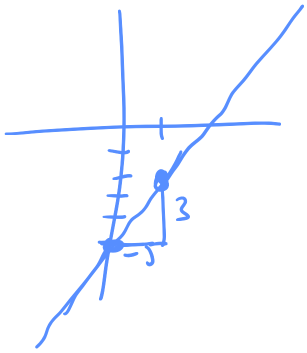
nacht

$$(5x)' = 5$$

$$[\cos(\omega t)\cdot\omega]' = \omega\cdot(-\sin(\omega t)\cdot\omega) = \underline{\underline{-\omega^2\cdot\sin(\omega t)}}$$

Geradengleichung $y = \boxed{k}x + \boxed{d}$

$y = \boxed{3}x \boxed{-5}$



$$y(x) = \sqrt{25-x^2} \quad \text{bei } x_0 = 4$$

$$y'(x_0) = ?$$

$$y(x) = (25-x^2)^{1/2}$$ ← äußere Fkt

$$y'(x) = \frac{1}{2}(25-x^2)^{\frac{1}{2}-1}\cdot(0-2x)$$

$$= \frac{1}{\cancel{2}}(25-x^2)^{-\frac{1}{2}}(-\cancel{2}x)$$

$$= \frac{-x}{(25-x^2)^{1/2}} = \underline{\underline{\frac{-x}{\sqrt{25-x^2}}}}$$

$$x^{-7} = \frac{1}{x^7}$$

$$y'(x_0) = y'(4) = \frac{-4}{\sqrt{25-4^2}} = \frac{-4}{3} = \underline{\underline{-\frac{4}{3}}}$$

$$y_{Tang} = -\frac{4}{3}x + d$$

2.

$$y(4) = \sqrt{25-4^2} = 3$$ Einsetzen

$$3 = -\frac{4}{3}\cdot 4 + d$$

$$3 + \frac{16}{3} = d, \quad d = \frac{9}{3} + \frac{16}{3} = \frac{25}{3}$$

$$\underline{\underline{y_{Tang} = -\frac{4}{3}\cdot x + \frac{25}{3}.}}$$

(5) $y(x) = \dfrac{x^2+1}{x-3}$. Definitionsmenge $= \mathbb{R}\setminus\{3\}$

Polstelle $= 3$

Nullstellen: keine

Bei welchem x ist $\frac{x^2+1}{x-3} \overset{?}{=} 0$ d.h. $x^2+1 \overset{?}{=} 0$

$x^2 = -1$ keine Lsg

Extremwerte: Minima u. Maxima

$$y'(x) = \left(\frac{x^2+1}{x-3}\right)' = \frac{2x\cdot(x-3)-(x^2+1)\cdot 1}{(x-3)^2} = \frac{2x^2-6x-x^2-1}{(x-3)^2}$$

$$= \frac{x^2-6x-1}{(x-3)^2} \overset{!}{=} 0$$

d.h. $x^2 \underbrace{-6}_{p} x \underbrace{-1}_{q} = 0$

$x^2+px+q=0$

$x_{1,2} = \underbrace{-\frac{p}{2}}_{} \pm \sqrt{\underbrace{\frac{p^2}{4}}_{} - q}$

$x_{1,2} = 3 \pm \sqrt{9+1}$

$x_1 = 3+\sqrt{10} = \underline{6{,}162}$

$x_2 = 3-\sqrt{10} = \underline{-0{,}162}$

(11) $f(x) = \frac{1}{x}$, $x_0 = 1$, $\frac{1}{x^1} = x^{-1}$

$f(x) = x^{-1}$

$f'(x) = -1\cdot x^{-1-1} = -x^{-2} = -\frac{1}{x^2}$

$f''(x) = -(-2)\cdot x^{-2-1} = 2x^{-3} = \frac{2}{x^3}$

$f(1) = 1$

$f'(1) = -1$

$f''(1) = 2$

$$f_{Parabel}(x) = f(1) + f'(1)\cdot(x-1) + \frac{1}{2}\cdot f''(1)\cdot(x-1)^2$$

$$= 1 + (-1)\cdot(x-1) + \frac{1}{\cancel{2}}\cdot\cancel{2}\cdot(x-1)^2$$

$|(a-b)^2 = a^2-2ab+b^2$

$$= 1 - x + 1 + x^2 - 2x + 1$$

$$= \underline{\underline{3 - 3x + x^2}}$$

(13) $\Phi(r) = -G\frac{mM_E}{r}$, $\frac{1}{r} = r^{-1}$

$= -GmM_E\cdot r^{-1}$

$\Phi'(r) = -GmM_E\cdot(-1)r^{-2} = \underbrace{G\frac{mM_E}{r^2}}$

$\Phi_{Tang}(r_E+h) = \Phi(r_E) + \underbrace{\Phi'(r_E)\cdot(\underbrace{r_E+h-r_E}_{h})}$

$g = 9{,}81\,\frac{m}{s^2}$

$\text{const} + \underline{\underline{G\frac{mM_E}{r_E^2}\cdot h}} = \text{const} + m\cdot g\cdot h$