R3)

1) $y = kx + d$

Finde k und d, um $\sum_{i=1}^{n} e_i^2$ zu minimieren, mit

$$e_1^2 = (\hat{y}_1 - y_1)^2$$

$$e_2^2 = (\hat{y}_2 - y_2)^2 \quad \text{usw.}$$

$$\left\| \begin{pmatrix} 1960 & 1 \\ 1965 & 1 \\ 1970 & 1 \\ 1975 & 1 \\ 1980 & 1 \\ 1985 & 1 \\ 1990 & 1 \\ 1995 & 1 \end{pmatrix} \cdot \begin{pmatrix} k \\ d \end{pmatrix} - \begin{pmatrix} 86 \\ 99{,}8 \\ 135{,}8 \\ 155 \\ 192{,}6 \\ 243{,}1 \\ 316{,}3 \\ 469{,}5 \end{pmatrix} \right\| \longrightarrow \min.$$

2) $y = c e^{\alpha t} \quad | \cdot \ln$

$\ln|y| = \alpha t + \ln|c|$ vergleiche $y = kt + d$ (mit $y \leftarrow \ln|y|$, $k \leftarrow \alpha$, $d \leftarrow \ln|c|$)

$$\left\| \begin{pmatrix} 1960 & 1 \\ 1965 & 1 \\ 1970 & 1 \\ 1975 & 1 \\ 1980 & 1 \\ 1985 & 1 \\ 1990 & 1 \\ 1995 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \ln(c) \end{pmatrix} - \begin{pmatrix} \ln 86 \\ \ln 99{,}8 \\ \ln 135{,}8 \\ \ln 155 \\ \ln 192{,}6 \\ \ln 243{,}1 \\ \ln 316{,}3 \\ \ln 469{,}5 \end{pmatrix} \right\| \longrightarrow \min.$$

MD 1)

$$T(x,t) = \frac{1}{\sqrt{t}} e^{-\frac{x^2}{4t}} \qquad \partial T/\partial t = \partial^2 T/\partial x^2$$

$$\partial T/\partial t = -\frac{1}{2} t^{-3/2} e^{-x^2/4t} + \frac{1}{\sqrt{t}} e^{-x^2/4t} \cdot \frac{x^2}{4t^2}$$

$$\partial T/\partial x = \frac{1}{\sqrt{t}} \cdot e^{-x^2/4t} \cdot (-2x/4t)$$

$$\partial^2 T/\partial x^2 = \frac{1}{\sqrt{t}} e^{-x^2/4t} \cdot (4x^2/16t^2) - \frac{1}{2} t^{-3/2} e^{-x^2/4t}$$

$$\partial T/\partial t = \partial^2 T/\partial x^2 \;\checkmark$$

MD2) 1) $f(x,y,z) = \begin{pmatrix} \cos(x-z) \\ 4z+xy \end{pmatrix}$

$$\underset{\sim}{J} = \begin{pmatrix} -\sin(x-z) & 0 & \sin(x-z) \\ y & x & 4 \end{pmatrix} = \begin{pmatrix} \partial f_x/\partial x & \partial f_x/\partial y & \partial f_x/\partial z \\ \partial f_y/\partial x & \partial f_y/\partial y & \partial f_z/\partial z \end{pmatrix}$$

2) $z(x,y) = e^{-(3x^2+y^2)}$

$$dz = -6x e^{-(3x^2+y^2)} dx - 2y e^{-(3x^2+y^2)} dy$$

$$dz\Big|_{x_0,y_0} = -6e^{-7} dx - 4e^{-7} dy$$

$$dz\Big|_{\substack{dx=0,1 \\ dy=-0,2}} = -6e^{-7}\, 0,1 + 4e^{-7}\, 0,2 = \underline{0,00018238}$$

$$\Delta z = z(1,1;1,8) - z(1;2) =$$

$$0,00103848 - 0,000911882 = \underline{0,0001266}$$

MD3) 1) $V = 0,08\, T/p$

$$\partial V/\partial p = -0,08\, T/p^2\Big|_{20,300} = \underline{-0,06}$$

$$\partial V/\partial T = 0,08\, 1/p\Big|_{20,300} = \underline{0,004}$$

Volumen steigt mit zunehmender ~~Druck~~ Temperatur und sinkt mit zunehmenden Druck!

MD4)

$$Q(x,y,z) = 15xz + 14yz + 11xy$$

$$Q(30,12,9) = \underline{9522\, W}$$

$$\partial Q/\partial x\Big|_{30,12,9} = 15z + 11y\Big|_{30,12,9} = \underline{267\, W/m}$$

$$\partial Q/\partial y\Big|_{30,12,9} = 14z + 11x\Big|_{30,12,9} = \underline{456\, W/m}$$

$$\partial Q/\partial z\Big|_{30,12,9} = 15x + 14y\Big|_{30,12,9} = \underline{618\, W/m}$$

→ Wärmeverlust reagiert am meisten auf eine Änderung der Höhe des Gebäudes!

MD5) $z(x,y) = \sqrt{x^2+4y^2}$

$T(x,y) = 100 + 2x - \frac{1}{4}x^2y^2$

1) $dz = \frac{1}{2\sqrt{x^2+4y^2}} \cdot 2x\,dx + \frac{1}{2\sqrt{x^2+4y^2}} 8y\,dy$

$dz|_{3,2} = \frac{6}{10}\,dx + \frac{16}{10}\,dy = \frac{3}{5}dx + \frac{8}{5}dy$

$dT = 2 - 0{,}5xy^2\,dx - 0{,}5x^2y\,dy$

$dT|_{3,2} = -4\,dx - 9\,dy$

2) $z - z(3,2) = \frac{3}{5}(x-3) + \frac{8}{5}(y-2)$

$z - 5 = \frac{3}{5}x - \frac{9}{5} + \frac{8}{5}y - \frac{16}{5}$ $\quad | \cdot 5$

$5z = 3x + 8y$

$z = \partial z/\partial x\,(x - x_0) + \partial z/\partial y\,(y-y_0) + z(x_0,y_0)$

3) ~~[illegible]~~

$dT = (\nabla T)^T \begin{pmatrix} dx \\ dy \end{pmatrix}$

4) ~~$\frac{dT}{dz} = \frac{\frac{3}{5}dx + \frac{8}{5}dy}{4dx - 9dy}$~~

$\nabla T = \begin{pmatrix} -4 \\ -9 \end{pmatrix}$ → Gradient gibt stärksten Anstieg!

Bewegung in Richtung des stärksten Temp.-anstiegs!

$\Rightarrow dx = -4;\ dy = -9$

$\frac{dT}{dz} = \frac{-4\,dx - 9\,dy}{0{,}6\,dx + 1{,}6\,dy} = \frac{-4\cdot(-4) - 9\cdot(-9)}{0{,}6\cdot(-4) + 1{,}6\,(-9)} =$

$-5{,}77$ F/km