MF1)

1) $$f(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \sqrt{t} \begin{pmatrix} \cos(\pi t) \\ \sin(\pi t) \end{pmatrix}$$

$$f'(t) = \begin{pmatrix} \frac{1}{2\sqrt{t}} \cos(\pi t) + \sqrt{t}\,(-\sin(\pi t)\pi) \\ \frac{1}{2\sqrt{t}} \sin(\pi t) + \sqrt{t} \cos(\pi t)\pi \end{pmatrix}$$

2) $$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{t\cos^2(\pi t) + t\sin^2(\pi t)} = \sqrt{t}$$

$$\varphi = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{\sin(\pi t)}{\cos(\pi t)}\right)$$

$$= \arctan(\tan(\pi t))$$

$$= \pi t$$

$$r'(t) = \frac{1}{2\sqrt{t}} \qquad \varphi'(t) = \pi$$

MF2)

1) $$F(x,y) = \frac{1}{x^2+y^2} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{2x^2 - (x^2+y^2)}{(x^2+y^2)^2} + \frac{2y^2 - (x^2+y^2)}{(x^2+y^2)^2}$$

$$= \frac{2x^2 - 2x^2 + 2y^2 - 2y^2}{(x^2+y^2)^2}$$

$$= 0$$

quellfrei ... einzige Quelle bei $x = 0, y = 0$, aber da ist die Funktion nicht definiert.

2) $$F(x,y,z) = \begin{pmatrix} xy - z^2 \\ 2xyz \\ x^2z - y^2z \end{pmatrix}$$

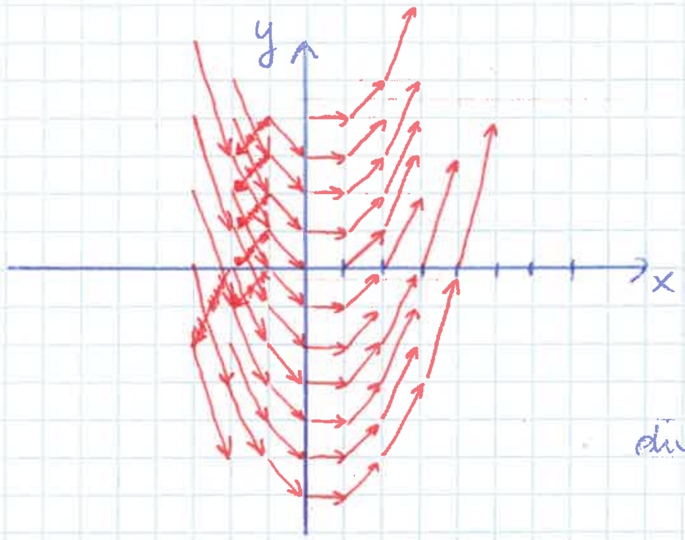
$$\operatorname{rot} F = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix} \times \begin{pmatrix} xy - z^2 \\ 2xyz \\ x^2z - y^2z \end{pmatrix} = \begin{pmatrix} -2yz - 2xy \\ -2xz - 2z \\ 2yz - x \end{pmatrix}$$

MF3)

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \sin(2t) \\ \cos(3t) \end{pmatrix}$$

$$T(t=1) = \begin{pmatrix} \sin 2 \\ \cos 3 \end{pmatrix} + \begin{pmatrix} 2\cos 2 \\ -3\sin 3 \end{pmatrix} \cdot t$$

MF4) 1) $F(x,y) = \begin{pmatrix} 1 \\ x \end{pmatrix}$



P	F
1,0	1,1
2,0	1,2
3,0	1,3
3,1	1,3
3,2	1,3
-2,5	1,-2
0,1	1,0
0,5	1,0

$\operatorname{div} F = \nabla \cdot F$

$= \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \end{pmatrix}^T \cdot \begin{pmatrix} 1 \\ x \end{pmatrix} = \underline{0}$

Vektorfeld ist quellfrei!