LA2-6)

$$\underset{\sim}{R} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

1) beschreibt eine Rotation, die mathematisch positiv (gegen den Uhrzeigersinn verläuft

2) $$\underset{\sim}{R}^T \underset{\sim}{R} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} \cos^2\alpha+\sin^2\alpha & 0 \\ 0 & \cos^2\alpha+\sin^2\alpha \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \underset{\sim}{I} \checkmark$$

$$\det \underset{\sim}{R} = \cos^2\alpha + \sin^2\alpha = \underline{1}$$

$$\underset{\sim}{R}^{-1} = \frac{1}{1} \cdot \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$

LA2-8)

1) $$\begin{pmatrix} -10 & 3 \\ -5 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3{,}5 \\ 9{,}2 \end{pmatrix}$$

$$Rg(A) = ? \quad \begin{pmatrix} -10 & 3 \\ -5 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} -10 & 3 \\ 0 & 2{,}5 \end{pmatrix}$$

$Rg(A) = 2 = Rg(A|b) = n$

$\Rightarrow$ eindeutig lösbar!

2) $$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}\vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad Rg(A) = 1 < Rg(A|b)$$

$\Rightarrow$ keine Lösung

3) $$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad Rg(A) = Rg(A|b) = 1 < n = 2$$

$\Rightarrow$ Lösungsraum $\mathbb{R}$

LA2-9)

1) $$\underset{\sim}{M} = \begin{pmatrix} 0{,}9 & 0{,}2 \\ 0{,}1 & 0{,}8 \end{pmatrix}$$

$$\vec{x_1} = \underset{\sim}{M}\, \vec{x_0}$$

2) $$\det(\underset{\sim}{M} - \lambda \underset{\sim}{I}) = \vec{0}$$

$$\det\begin{pmatrix} 0{,}9-\lambda & 0{,}2 \\ 0{,}1 & 0{,}8-\lambda \end{pmatrix} = (0{,}9-\lambda)(0{,}8-\lambda) - 0{,}02 = 0$$

$$0{,}72 - 1{,}7\lambda + \lambda^2 - 0{,}02 = 0$$

$$\lambda^2 - 1{,}7\lambda + 0{,}7 = 0$$

$$\lambda_{1/2} = 0{,}85 \pm \sqrt{0{,}0225}$$

$$\Rightarrow \lambda_1 = 1 ;\ \lambda_2 = 0{,}7$$

EV zu $\lambda = 1$

$$\begin{pmatrix} -0{,}1 & 0{,}2 \\ 0{,}1 & -0{,}2 \end{pmatrix} \vec{x} = \vec{0}$$

$$0{,}1 x_1 = 0{,}2 x_2$$

$$x_1 = 2x_2$$

$$\text{EV}: \begin{pmatrix} 2 \\ 1 \end{pmatrix}\mathbb{R} \qquad \text{z.B.}: \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1{,}5 \end{pmatrix}, \dots$$

$$x_{\infty 1} + x_{\infty 2} = 18 (\text{Mio.})$$

$$x_{\infty 1} = 2 x_{\infty 2}$$

$$\Rightarrow 3 x_{\infty 2} = 18$$

$$x_{\infty 2} = 6 \Rightarrow x_{\infty 1} = 12$$

$$\vec{x_\infty} = \begin{pmatrix} 12 \\ 6 \end{pmatrix}$$

## LA2-10)

$$\det\begin{pmatrix} 2 & -1 & -1 \\ 1 & 2 & 2 \\ 0 & 5 & -1 \end{pmatrix}\begin{matrix} 2 & -1 \\ 1 & 2 \\ 0 & 5 \end{matrix} = -4 - 5 - 20 - 1$$

$$= -30 \neq 0$$

$\Rightarrow$ linear unabhängig

## LA2-11)

$$\underset{\sim}{A} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \qquad \underset{\sim}{A}^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \checkmark$$

$$\underset{\sim}{A}^{-1} = \frac{1}{\underset{\det \underset{\sim}{A}}{1}} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \checkmark$$

## LA2-12)

$$\underset{\sim}{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

$$\det\begin{pmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 0 \\ 0 & 1 & 3-\lambda \end{pmatrix}\begin{matrix} 1-\lambda & 0 \\ 0 & 2-\lambda \\ 0 & 1 \end{matrix} =$$

$$(1-\lambda)(2-\lambda)(3-\lambda) = 0$$

$$\Rightarrow \lambda_1 = 1;\ \lambda_2 = 2;\ \lambda_3 = 3$$

$EV_1: \begin{pmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}\vec{x} = 0 \qquad x_2 = 0;\ x_3 = 0;\ x_1 = \mathbb{R} \qquad EV: \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\mathbb{R}$

$EV_2: \begin{pmatrix} -1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}\vec{x} = 0 \qquad x_1 = -2x_2;\ x_3 = -x_2 \qquad EV: \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}\mathbb{R}$

$EV_3: \begin{pmatrix} -2 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix}\vec{x} = 0 \qquad x_1 = x_3;\ x_2 = 0 \qquad EV: \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\mathbb{R}$

2) Ja, ist invertierbar, weil $\det A \neq 0$

3) Ja, eindeutig lösbar!

LA2-13)

$$\begin{pmatrix}-10 & 3\\ -5 & 4\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}-3{,}5\\ 9{,}2\end{pmatrix}$$

$\cancel{k}$ $\det(A) = -40+15 \neq 0 \Rightarrow$

eindeutig lösbar!

LA2-14)

$$\det\begin{pmatrix}1 & -3 & 2\\ -2 & 7 & 1\\ -4 & 6 & h\end{pmatrix}\begin{matrix}1 & -3\\ -2 & 7\\ -4 & 6\end{matrix} =$$

$$7h+12-24+28\cdot 2-6-6h=0$$

$$h+38=0$$

$$h=-38$$

LA2-15)

$$\det\begin{pmatrix}2-\lambda & 7\\ 7 & 2-\lambda\end{pmatrix}=$$

$$4-4\lambda+\lambda^2-49=0$$

$$\lambda^2-4\lambda-45=0$$

$$\lambda_{1/2}=2\pm\sqrt{49} \nearrow \lambda_1=9, \searrow \lambda_2=-5$$

$EV_1$: $\begin{pmatrix}-7 & 7\\ 7 & -7\end{pmatrix}\vec{x}=0$ $\quad EV: \begin{pmatrix}+1\\ +1\end{pmatrix}\mathbb{R}$

$EV_2$: $\begin{pmatrix}7 & 7\\ 7 & 7\end{pmatrix}\vec{x}=0$ $\quad EV: \begin{pmatrix}1\\ -1\end{pmatrix}\mathbb{R}$

LA2-16)

$$\det\begin{pmatrix}-2-\lambda & -5\\ 1 & 4-\lambda\end{pmatrix}=(-2-\lambda)(4-\lambda)+5=0$$

$$-3-2\lambda+\lambda^2=0$$

$$\lambda_{1/2}=1\pm\sqrt{4} \nearrow \lambda_1=3, \searrow \lambda_2=-1$$

$EV_1$: $\begin{pmatrix}-5 & -5\\ 1 & 1\end{pmatrix}\vec{x}=0$ $\quad EV: \begin{pmatrix}1\\ -1\end{pmatrix}\mathbb{R}$

$EV_2$: $\begin{pmatrix}-1 & -5\\ 1 & 5\end{pmatrix}\vec{x}=0$ $\quad EV: \begin{pmatrix}5\\ -1\end{pmatrix}\mathbb{R}$