LA1-1)

$$\vec{a}_1 = \begin{pmatrix}1\\2\\3\end{pmatrix}, \vec{a}_2 = \begin{pmatrix}4\\5\\6\end{pmatrix}, \vec{a}_3 = \begin{pmatrix}2\\1\\0\end{pmatrix}$$

a) z.B.: $\det\begin{pmatrix}1&4&2\\2&5&1\\3&6&0\end{pmatrix}\begin{matrix}1&4\\2&5\\3&6\end{matrix} = 12+24-30-6 = \underline{0}$

$\Rightarrow$ linear abhängig

b) $\begin{pmatrix}1&4&2\\2&5&1\\3&6&0\end{pmatrix}\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{pmatrix} = \vec{0}$

$$\left(\begin{array}{ccc|c}1&4&2&0\\2&5&1&0\\3&6&0&0\end{array}\right) \rightarrow \left(\begin{array}{ccc|c}1&4&2&0\\0&-3&-3&0\\0&-6&-6&0\end{array}\right) \rightarrow \left(\begin{array}{ccc|c}1&4&2&0\\0&-3&-3&0\\0&0&0&0\end{array}\right)$$

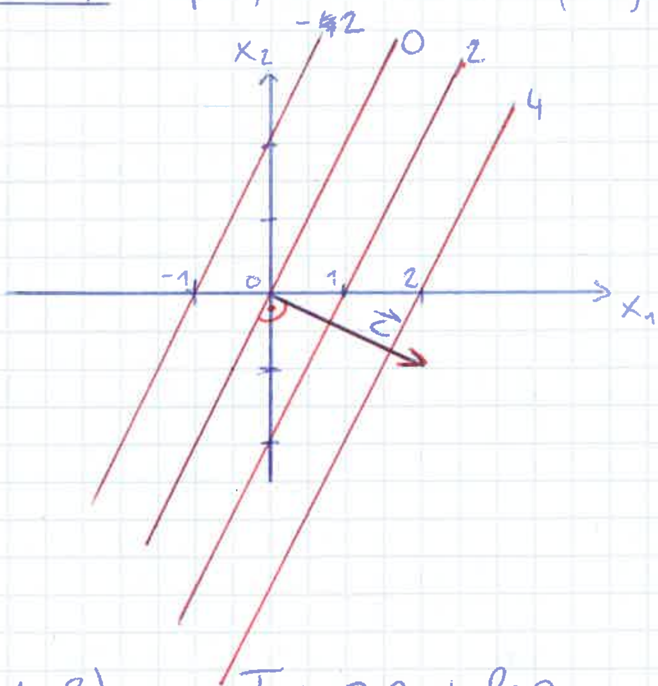
z.B.: $\underline{\alpha_3 = 1 \Rightarrow \alpha_2 = -1}$

$\alpha_1 + 4\alpha_2 + 2\alpha_3 = 0 \Rightarrow \underline{\alpha_1 = 2}$

$\underline{\alpha_3 \text{ wäre frei wählbar}}$

LA1-2)

$f(x) = 2x_1 - x_2 = \begin{pmatrix}2\\-1\end{pmatrix}^T \begin{pmatrix}x_1\\x_2\end{pmatrix}$



LA1-3)

$T = a p_1 + b p_2$

$30 = 2a + 2b$

$60 = 8a + 2b$

$\Rightarrow \begin{pmatrix}2&2\\8&2\end{pmatrix} \cdot \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}30\\60\end{pmatrix}$

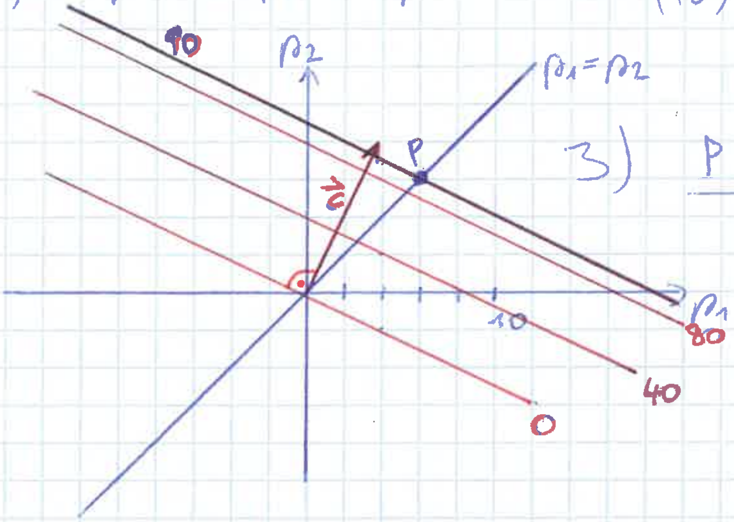
$\begin{pmatrix}2&2\\0&-6\end{pmatrix} \cdot \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}30\\-60\end{pmatrix}$

$\underline{T = 5p_1 + 10p_2}$

$\underline{\Rightarrow b = 10}$

$\underline{\Rightarrow a = 5}$

2) $T(\vec{p}) = 5p_1 + 10p_2 \quad \Rightarrow \vec{c} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$



3) $\underline{P = (6,6)}$

LA1-4) 1) Kapitalwert: Zeitwert des Geldes (Kapitalfluss), auf den aktuellen Zeitpunkt abgezinst.

2) $K = \frac{-18}{1{,}05^0} + \frac{1}{1{,}05^1} + \frac{1}{1{,}05^2} + \dots + \frac{21}{1{,}05^{10}}$

$K(5\%) = \underline{2}$

$K(10\%) = \underline{-4{,}14}$

3) $f(c) = \frac{c}{(1+i)^t}$

c ... cash flow

i ... Zinssatz

t ... Zeitpunkt des cash flows

4) Ja, weil f(c) eine lineare Funktion ist.

LA2-1)

$$\begin{pmatrix}2&-5&8\\-2&-7&1\\4&2&7\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\vec{0}$$

$$\left(\begin{array}{ccc|c}2&-5&8&0\\-2&-7&1&0\\4&2&7&0\end{array}\right)\longrightarrow\left(\begin{array}{ccc|c}2&-5&8&0\\0&-12&9&0\\0&12&-9&0\end{array}\right)\longrightarrow$$

$$\longrightarrow\left(\begin{array}{ccc|c}2&-5&8&0\\0&-12&9&0\\0&0&0&0\end{array}\right)\Rightarrow \infty \text{ Lösungen}$$

LA2-2)

$$\det\begin{pmatrix}2&-4&-2\\-5&1&1\\7&-5&-3\end{pmatrix}\begin{matrix}2&-4\\-5&1\\7&-5\end{matrix}=-6-28-50$$

$$-[-14-10-60]$$

$$=-84+84=\underline{0}$$

$\Rightarrow$ linear abhängig

$\Rightarrow \underline{Rg(A)<n}$

$\Rightarrow$ in Abhängigkeit von $\vec{b}$ keine oder ∞ Lösungen!

LA2-3)

$C: \ 3x_1 = x_3$ ✓

$H: \ 8x_1 = 2x_4$ ✓

$O: \ 2x_2 = 2x_3 + x_4$ ✓

$$x_1\begin{pmatrix}3\\8\\0\end{pmatrix}+x_2\begin{pmatrix}0\\0\\2\end{pmatrix}=x_3\begin{pmatrix}1\\0\\2\end{pmatrix}+x_4\begin{pmatrix}0\\2\\1\end{pmatrix} ✓$$

$$\begin{pmatrix}3&0&-1&0\\8&0&0&-2\\0&2&-2&-1\end{pmatrix}\cdot\vec{x}=\vec{0}$$

$$\left(\begin{array}{cccc|c}3&0&-1&0&0\\0&0&8/3&-2&0\\0&2&-2&-1&0\end{array}\right)$$

$$\left(\begin{array}{cccc|c}3&0&-1&0&0\\0&2&-2&-1&0\\0&0&8/3&-2&0\end{array}\right)$$

$\underline{\mathcal{L}=\mathbb{R}}$

z.B.: $x_4 = \mathbb{R}$

$8/3\, x_3 - 2x_4 = 0$

$\Rightarrow \underline{x_3 = 0{,}75 x_4}$

$2x_2 - 1{,}5x_4 - x_4 = 0$

$\underline{x_2 = 1{,}25 x_4}$

$3x_1 - 0{,}75 x_4 = 0$

$\underline{x_1 = 0{,}25 x_4}$

kleinste Lösung: $\underline{\vec{x}=\begin{pmatrix}1\\5\\3\\4\end{pmatrix}}$

LA2-4)

1)

① $I_1 + I_3 - I_2 = 0$

② $U_1 = I_1 R_1 + I_2 R_2$

③ $U_2 = I_3 R_3 + I_2 R_2$

$$\Rightarrow \begin{pmatrix} 1 & -1 & 1 \\ R_1 & R_2 & 0 \\ 0 & R_2 & R_3 \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0 \\ U_1 \\ U_2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 200 & 100 & 0 & 230 \\ 0 & 100 & 300 & 370 \end{array}\right)$$

2)

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 300 & -200 & 230 \\ 0 & 100 & 300 & 370 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 300 & -200 & 230 \\ 0 & 0 & \frac{1100}{3} & \frac{880}{3} \end{array}\right)$$

$1100 I_3 = 880$

$\underline{I_3 = 0{,}8}$

$300 I_2 - 200 I_3 = 230$

$300 I_2 = 390$

$\underline{I_2 = 1{,}3} \Rightarrow \underline{I_1 = 0{,}5}$

LA2-5)

$$\underset{\sim}{A}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\underset{\sim}{A}^{-1} \underset{\sim}{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \text{erfüllt?}$$

$$\frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} =$$

$$\frac{1}{ad - bc} \cdot \begin{pmatrix} ad - bc & 0 \\ 0 & -bc + ad \end{pmatrix} = \underline{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} \checkmark$$

$$\underset{\sim}{A} = \begin{pmatrix} 0{,}5 & 0 \\ 0 & 3 \end{pmatrix} \qquad \det A = \underline{1{,}5}$$

$$A^{-1} = \frac{1}{1{,}5} \cdot \begin{pmatrix} 3 & 0 \\ 0 & 0{,}5 \end{pmatrix} = \underline{\begin{pmatrix} 2 & 0 \\ 0 & 1/3 \end{pmatrix}}$$