AI4)

1) $dS = \frac{C_V}{T}\,dT + \frac{R}{V}\,dV$

$$\begin{pmatrix} \partial/\partial T \\ \partial/\partial V \end{pmatrix} \times \begin{pmatrix} C_V/T \\ R/V \end{pmatrix} = 0 \checkmark \Rightarrow \text{exakt}$$

① $V = dV = 0$

$$\int_0^T \frac{C_V}{T}\,dT = C_V \ln[T] + C$$

② $dT = 0$

$$\int_0^V \frac{R}{V}\,dV = R\ln[V]$$

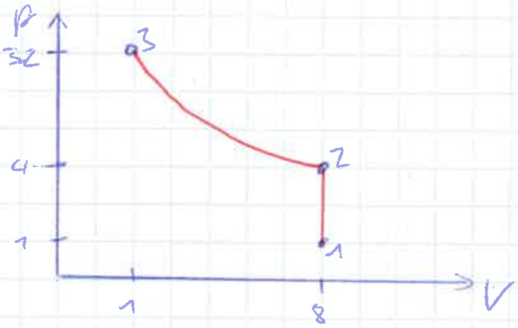
$$S(T,V) = c_V \ln T + R\ln V + C$$

$$S(T,V) = \frac{3}{2}R\ln\left(\frac{pV}{R}\right) + R\ln V + C$$

$$= \frac{3}{2}R\ln(pV) + \frac{3}{2}R\ln\left(\frac{1}{R}\right) + \frac{3}{2}R\ln\left(V^{2/3}\right) + C$$

$$= \frac{3}{2}R\ln\left(pV^{5/3}\right)\underbrace{\left(+\frac{3}{2}R\ln\left(\frac{1}{R}\right) + C\right)}_{\text{konstant}}$$

AI5)



$pV = RT$

$$T = \frac{pV}{R} = \frac{4 \cdot 8}{R} = \frac{32}{R}$$

$$W_{V_{12}} = +\int_1^2 p\,dV = 0 \qquad Q_{12} = \int \frac{5}{2}p\,\overset{0}{\cancel{dV}} + \frac{3}{2}V\,dp$$

$$= \frac{3}{2}V(p_2 - p_1)$$

$$= \frac{3}{2}\cdot 8 \cdot (3) = 36$$

$$W_{V_{23}} = \int_2^3 p\,dV = \text{const.}\int_2^3 \frac{dV}{V} =$$

$$p_2V_2 \ln\left[\frac{V_3}{V_2}\right] = 32\ln\left[\frac{1}{8}\right] = -32\ln[8]$$

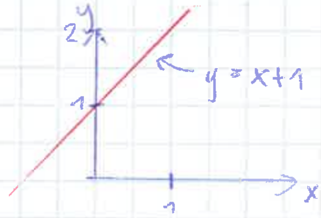
$$Q_{23} = \int_2^3 \frac{5}{2}p\,dV + \frac{3}{2}V\,dp \qquad p = \frac{32}{V} \qquad dp = -\frac{32}{V^2}\,dV$$

$$= \int_2^3 \frac{5}{2}\cdot\frac{32}{V}\,dV + \int_2^3 \frac{3}{2}V\cdot\left(-\frac{32}{V^2}\right)dV = 32\cdot\int_2^3 \frac{dV}{V} = 32\ln\left[\frac{V_3}{V_2}\right]$$

$$= -32\ln[8]$$

A16)

① $\int_{0,1}^{1,2} (x^2-y)\,dx + (y^2+x)\,dy$



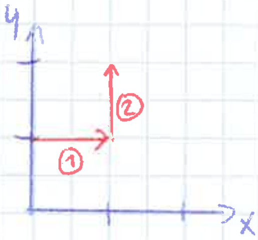
$y = x+1$

$dy = dx$

$\Rightarrow \int_0^1 (x^2 - x - 1 + x^2 + 2x + 1 + x)\,dx =$

$= \int_0^1 2x^2 + 2x\,dx = \frac{2}{3}x^3 + x^2 \Big|_0^1 = \underline{\frac{5}{3}}$

②



① $y = 1 \quad dy = 0$

$\int_0^1 (x^2 - 1)\,dx =$

$\frac{x^3}{3} - x \Big|_0^1 = \underline{-\frac{2}{3}}$

② $x = 1, \; dx = 0$

$\int_1^2 (y^2+1)\,dy = \left(\frac{y^3}{3} + y\right)\Big|_1^2 =$

$= \frac{8}{3} + 2 - \frac{1}{3} - 1 = \underline{10/3}$

A17)

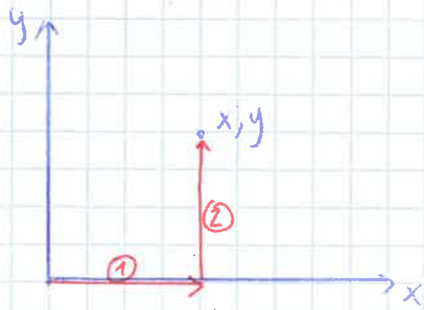
$(3y^2 - 12x)\,dx + (6xy+1)\,dy$

$\begin{pmatrix} \partial/\partial x \\ \partial/\partial y \end{pmatrix} \times \begin{pmatrix} 3y^2 - 12x \\ 6xy+1 \end{pmatrix} = 6y - 6y = 0$ ✓ exakt

$\Rightarrow$ es gibt Stammfunktion

① $y = dy = 0$

$\int_0^x -12x\,dx = \underline{-6x^2}$



② $x = x, \; dx = 0$

$\int_0^y (6xy+1)\,dy =$

$\underline{3xy^2 + y + C}$

$\underline{F(x,y) = 3xy^2 + y - 6x^2 + C}$