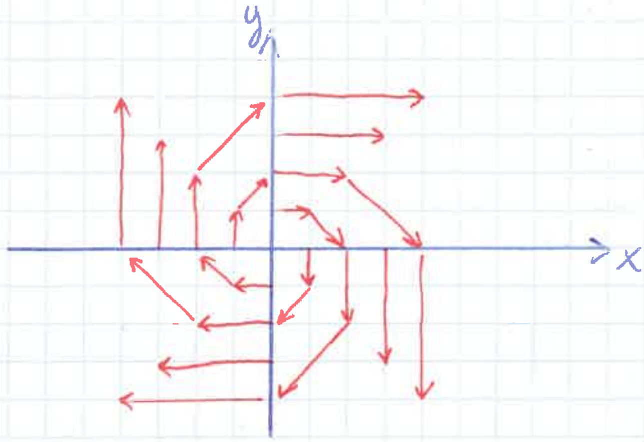
AI1)

$$\delta f = y\,dx - x\,dy$$

1) $\vec{F} = \begin{pmatrix} y \\ -x \end{pmatrix}$



2) rot $\vec{F} = 0$?

$$\begin{pmatrix} \partial/\partial x \\ \partial/\partial y \end{pmatrix} \times \begin{pmatrix} y \\ -x \end{pmatrix} = -1 - 1 = \underline{-2} \neq 0$$

$\Rightarrow$ <u>inexaktes Differential</u>

<u>wegabhängiges Vektorfeld</u>

3) $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} -\cos(t) \\ \sin(t) \end{pmatrix} \qquad 0 \le t \le \pi$

$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix} \Rightarrow \begin{array}{l} dx = \sin(t)\,dt \\ dy = \cos(t)\,dt \end{array}$

$$\int y\,dx - x\,dy =$$

$$\int [\sin(t) \cdot \sin(t) - (-\cos(t))\cos(t)]\,dt$$

$$\int (\sin^2(t) + \cos^2(t))\,dt = \int_0^{\pi} 1\,dt = \underline{\pi}$$

4) $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} t \\ 0 \end{pmatrix} \qquad -1 \le t \le 1$

$\dot{x} = 1 \Rightarrow dx = dt;\ dy = 0$

$$\int 0 \cdot dx - t \cdot dy^{\,0} = \int 0 = \underline{0}$$

Warum 0? Kraftvektoren stehen immer normal auf Wegrichtung!

AI 2)

$$F = \begin{pmatrix} xy \\ 1 \\ yz \end{pmatrix} \qquad \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} \cos(t) \\ \sin(t) \\ t \end{pmatrix} \qquad 0 \le t \le 2\pi$$

$$\delta f = \delta W = xy\,dx + dy + yz\,dz$$

$$\begin{pmatrix} \dot x \\ \dot y \\ \dot z \end{pmatrix} = \begin{pmatrix} -\sin(t) \\ \cos(t) \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} dx = -\sin(t)\,dt \\ dy = \cos(t)\,dt \\ dz = dt \end{matrix}$$

$$\int \delta W = \int \cos(t)\sin(t)(-\sin(t))\,dt + \cos(t)\,dt + \sin(t)\cdot t\,dt$$

$$= \int \cos(t)\cdot[1 - \sin^2(t)]\,dt + t\cdot\sin(t)\,dt =$$

$$= \int [\cos^3(t) + t\cdot\sin(t)]\,dt =$$

→ WOLFRAMALPHA

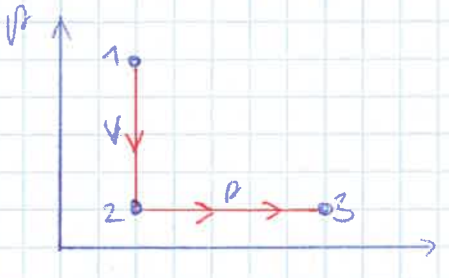
$$\int \cos^3(t)\,dt = \frac{1}{12}(9\sin(t) + \sin(3t))\Big|_0^{2\pi} = 0$$

$$= \int \underset{u}{t}\cdot\underset{v'}{\sin(t)}\,dt = -t\cdot\cos(t) - \int \cos(t)\,dt$$

$$= -t\cdot\cos(t) - \cancel{\sin(t)}\Big|_0^{2\pi} = -2\pi$$

AI 3)

1)



$$W_{12} = -\int p\,dV = 0$$

$$Q_{12} = \int T\,dS$$

$$S = \frac{3}{2} R \ln(pV^{5/3})$$

$$dS = \frac{3}{2} R\cdot\left[\frac{V^{5/3}}{pV^{5/3}}\,dp + \frac{p\frac{5}{3}V^{2/3}}{pV^{5/3}}\,dV\right]$$

$$= \frac{3}{2} R\left[\frac{dp}{p} + \frac{5\,dV}{3V}\right]$$

$$Q_{12} = \int_{32}^{1} \frac{\cancel{p}V}{\cancel{R}}\,\frac{3}{2}\cancel{R}\,\frac{dp}{\cancel{p}} = \frac{3}{2}Vp\Big|_{32}^{1}$$

$$= \frac{3}{2}\cdot 1\cdot[1 - 32] = -\frac{3}{2}\cdot 31$$

$$Q_{12} + W_{12} = -\frac{3}{2}\cdot 31 = U_2 - U_1 = \frac{3}{2}p_2V_2 - \frac{3}{2}p_2V_2$$

$$= \frac{3}{2}\cdot 1\cdot 1 - \frac{3}{2}\cdot 32\cdot 1 = -\frac{3}{2}\cdot 31 \checkmark$$

$Q_{23} = ?$ $\quad W_{23} = ?$

$$W_{23} = -\int p\,dV = -p(V_3 - V_2) = -1\cdot(8-1) = -7$$

$$Q_{23} = \int T\,dS$$

$$= \int \frac{p\cancel{V}}{\cancel{R}}\cdot\frac{\cancel{3}}{2}\cdot\frac{5}{\cancel{3}}\cancel{R}\,\frac{dV}{\cancel{V}} =$$

$$= \int \frac{5}{2}p\,dV = \frac{5}{2}p(V_3 - V_2) = \frac{5}{2}\cdot(8-1) = \frac{5}{2}\cdot 7$$

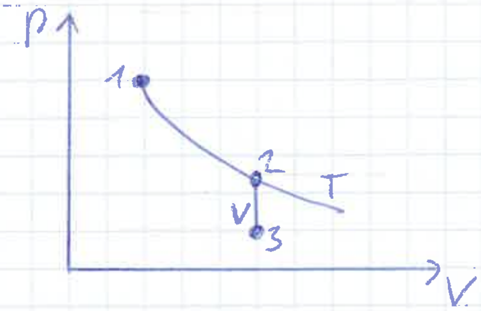
$$U_3 - U_2 = Q_{23} + W_{23}$$

$$\Rightarrow -7 + 7\cdot 5/2 = \frac{3}{2}p_3V_3 - \frac{3}{2}p_2V_2 =$$

$$\frac{3}{2}\cdot 1\cdot 8 - \frac{3}{2}\cdot 1\cdot 1$$

$$-\frac{14}{2} + \frac{35}{2} = \frac{21}{2} \checkmark$$

2)



$$W_{12} = -\int p\,dV$$

$$pV = \text{const.} = 32\cdot 1 = 32$$

$$p = \frac{32}{V}$$

$$W_{12} = -\int \frac{32}{V}dV =$$

$$-32\ln\left(\frac{V_2}{V_1}\right) = -32\ln(8)$$

$$Q_{12} = \int T\,dS = \int \frac{pV}{R}\frac{3}{2}R\left[\frac{dp}{p} + \frac{5}{3}\frac{dV}{V}\right]$$

$$= \int 32\cdot\frac{3}{2}\cdot\left[-\frac{32}{\cancel{V^2\,32/V}}dV + \frac{5}{3}\frac{dV}{V}\right]$$

$$= \int 32\cdot\frac{3}{2}\left[\frac{2}{3}\frac{dV}{V}\right]$$

$$= 32\ln\left[\frac{V_2}{V_1}\right]\Big|_1^8 = 32\ln(8)$$

$$\frac{dp}{dV} = -\frac{32}{V^2}$$

$$dp = -\frac{32}{V^2}dV$$

$$pV = 32$$

$$p_2 = p_1\left(\frac{V_1}{V_2}\right) =$$

$$32\cdot\left(\frac{1}{8}\right) = 4$$

$$U_2 - U_1 = 0 = Q_{12} + W_{12} \checkmark$$

$$W_{23} = -\int p\,dV = 0$$

$$Q_{23} = \int T\,dS = \int \frac{pV}{\cancel{R}}\frac{3}{2}\cancel{R}\frac{dp}{\cancel{p}} = \int \frac{3}{2}V\,dp = \frac{3}{2}Vp\Big|_4^1$$

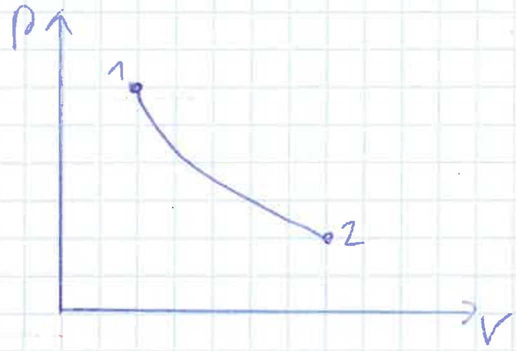
$$= \frac{3}{2}\cdot 8\cdot(-3)$$

$$= -36$$

$$U_3 - U_2 = \frac{3}{2}p_3V_3 - \frac{3}{2}p_2V_2 =$$

$$\frac{3}{2}\cdot[1\cdot 8 - 4\cdot 8] = -36$$

3)



$Q_{12} = 0$ (adiabatisch)

$dS = 0 \Rightarrow pV^{5/3} = \text{const.}$

$$W = -\int p\,dV = -\int \frac{c}{V^{5/3}}\,dV = -\frac{3}{2}\frac{c}{V^{2/3}}\Big|_1^8$$

$$= -\frac{3}{2}\cdot 32\cdot\left[\frac{1}{8^{2/3}} - \frac{1}{1}\right]$$

$$= -\frac{3}{2}\,32\cdot -\frac{3}{4}$$

$$= \underline{36}$$

$$U_2 - U_1 = \frac{3}{2}p_2V_2 - \frac{3}{2}p_1V_1 =$$

$$\frac{3}{2}[8\cdot 1 - 1\cdot 32] = \underline{36}$$