ADG1) 1) $y(x) = C \frac{x}{1+x}$ $\qquad y' = C \cdot \frac{-x+(1+x)}{(1+x)^2} = +C \frac{1}{(1+x)^2}$

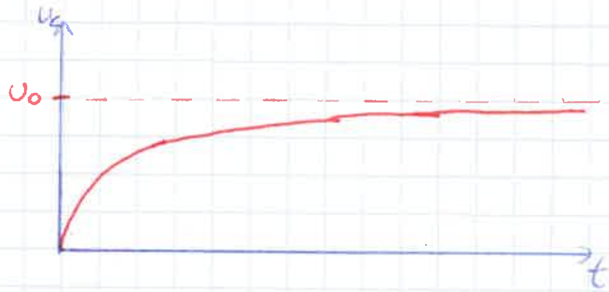
$$x(1+x)\,y' - y = 0$$

$$x\cancel{(1+x)} \cdot (+C) \frac{1}{(1+x)^{\cancel{2}}} - C \frac{1 \cdot x}{1+x} = 0 \checkmark$$

$$y(1) = 8 \Rightarrow C \cdot \frac{1}{2} = 8 \Rightarrow C = 16$$

$$\underline{y(x) = 16 \frac{x}{1+x}}$$

2) $u_c(1) = u_0 (1 - e^{-t/RC})$



$$RC\,\dot{u}_c(t) + u_c(t) = u_0$$

$$\dot{u}_c(t) + \frac{1}{RC} u_c(1) = \frac{1}{RC} \cdot u_0$$

vgl. $\dot{x} + a x = b$

$$x_\infty = \frac{b}{a} \Rightarrow u_{c,\infty} = \frac{u_0 \frac{1}{RC}}{\frac{1}{RC}} = u_0$$

$$x(1) = x_0 e^{-t \cdot a} + x_\infty (1 - e^{-t \cdot a})$$

$$\Rightarrow \underline{u_c(t) = u_0 (1 - e^{-t/RC})}$$

ADG3) 1)

$$y\dot{y} = te^t$$

$$y \frac{dy}{dt} = te^t$$

$$y\,dy = te^t\,dt \quad | \cdot \int$$

$$\frac{y^2}{2} = e^t(t-1) + C$$

$$\underline{y = \pm\sqrt{2e^t(t-1) + C_2}}$$

$\int t e^t dt$

→ partielle Integration

$\int \underset{u}{t} \cdot \underset{v'}{e^t} dt =$

$= t \cdot e^t - \int e^t dt =$

$t e^t - e^t = \underline{e^t \cdot (t-1)}$

AbG3) 2.)

$$\dot{y}(1+t^2) = ty$$
$$dy/dt = ty/(1+t^2)$$
$$\frac{dy}{y} = \frac{t}{(1+t^2)}\,dt \quad \Big|\cdot\int$$
$$\ln|y| = \ln|\sqrt{1+t^2}| + c$$
$$y(t) = e^{\ln\sqrt{1+t^2}+c}$$
$$\underline{y(t) = C\cdot\sqrt{1+t^2}}$$

$$\int \frac{t}{1+t^2}\,dt$$
$\rightarrow$ Substitution
$$z = 1+t^2$$
$$\frac{dz}{dt} = 2t \Rightarrow dt = \frac{dz}{2t}$$
$$\int \frac{\cancel{t}}{z}\,\frac{dz}{2\cancel{t}} =$$
$$0{,}5\ln(z) =$$
$$\underline{\ln|\sqrt{1+t^2}|}$$

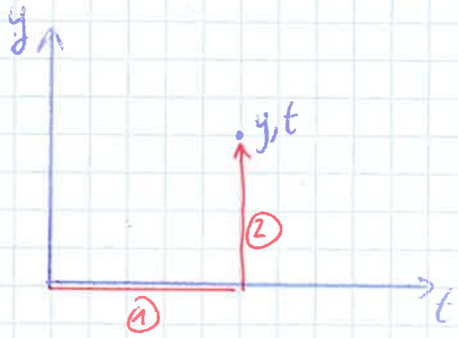
AbG4)

$$\dot{y} = \frac{-y^2}{2yt+1} \qquad y(1) = -2$$
$$\frac{dy}{dt} = \frac{-y^2}{2yt+1}$$
$$dF = (2yt+1)\,dy + y^2\,dt = 0$$

$$\begin{pmatrix}\partial/\partial y\\ \partial/\partial t\end{pmatrix}\times\begin{pmatrix}2yt+1\\ y^2\end{pmatrix} =$$
$$2y - 2y = 0 \checkmark$$
$\Rightarrow$ exakt

(1) $y = dy = 0$
$\Rightarrow \int df = 0$

(2) $t = t,\ dt = 0$
$\Rightarrow \int_0^y 2yt+1\,dy$



$$F(y,t) = \underline{y^2t + y + c}$$

$$F(y,t) = C_2, \text{ weil } dF = 0$$
$$\Rightarrow y^2t + y + c = C_2$$
$$y^2 + \frac{1}{t}y + C_3/t = 0$$
$$y_{1/2} = \frac{1}{2t} \pm \sqrt{\frac{1}{4t^2} - \frac{C_3}{t}}$$

$$y(1) = -2 \Rightarrow \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{C_3}{2}} = -2$$
$$\sqrt{\frac{1}{4} - \frac{C_3}{2}} = -\frac{5}{2}$$
$$\frac{1}{4} - \frac{C_3}{2} = \frac{25}{4}$$
$$-\frac{C_3}{2} = 6 \Rightarrow \underline{C_3 = -12}$$

$$\underline{y(t) = \frac{1}{2t} - \sqrt{\frac{1}{4t^2} + \frac{12}{t}}}$$